Imprint of the dark components on the CMB and LSS: part II

Thibaut Louis

The matter power spectrum

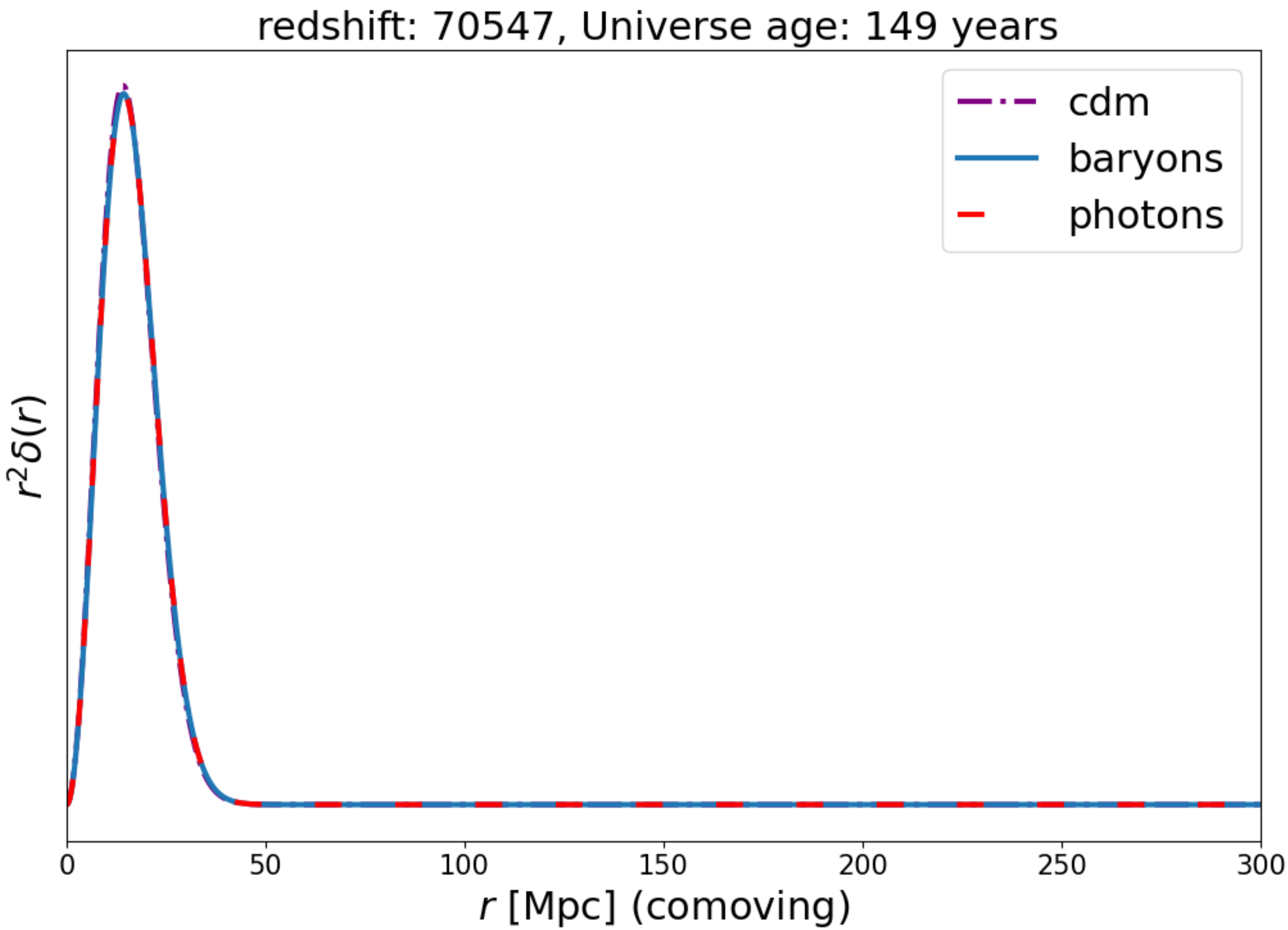
Last time, we discussed the Boltzmann equations which are the evolution equations for the macroscopic quantities describing the different physical species in our Universe

$$
\begin{aligned}
\rho_{\rm cdm}(\boldsymbol{x},t) &= \bar{\rho}_{\rm cdm}(t)(1+\delta_{\rm cdm}(\boldsymbol{x},t)) \\
\rho_{\rm b}(\boldsymbol{x},t) &= \bar{\rho}_{\rm b}(t)(1+\delta_{\rm b}(\boldsymbol{x},t)) \\
v_{\rm b}(\boldsymbol{x},t) & \\
v_{\rm cdm}(\boldsymbol{x},t) &
\end{aligned}
$$

And the Einstein equations that governs the evolution of the gravitational potential wells

$$
ds^2 = -(1+2\psi(\boldsymbol{x},t))dt^2 + a^2(t)\delta_{ij}[1+2\phi(\boldsymbol{x},t)]dx^i dx^j
$$

we also looked at the evolution of a single perturbation in the universe



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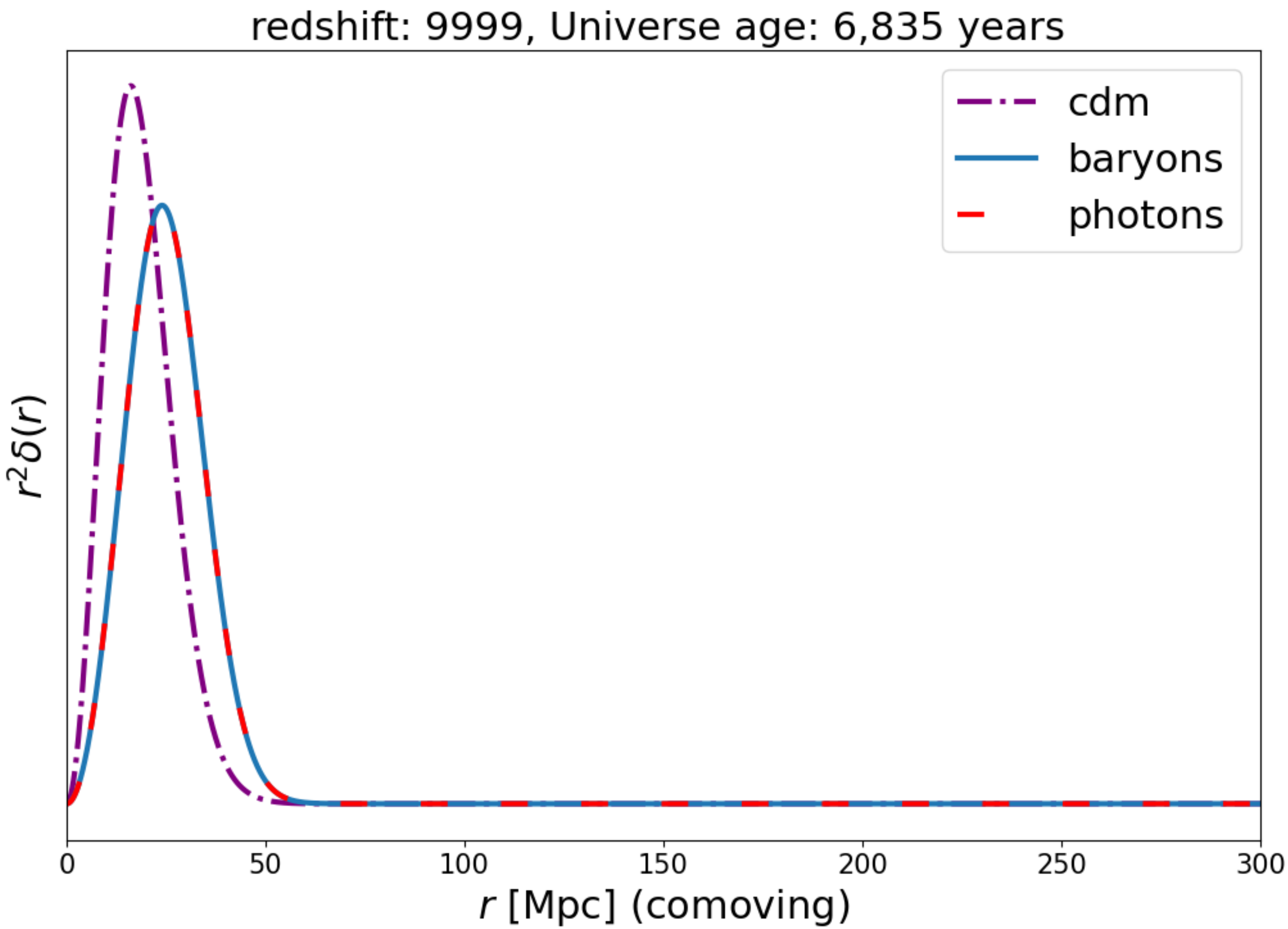

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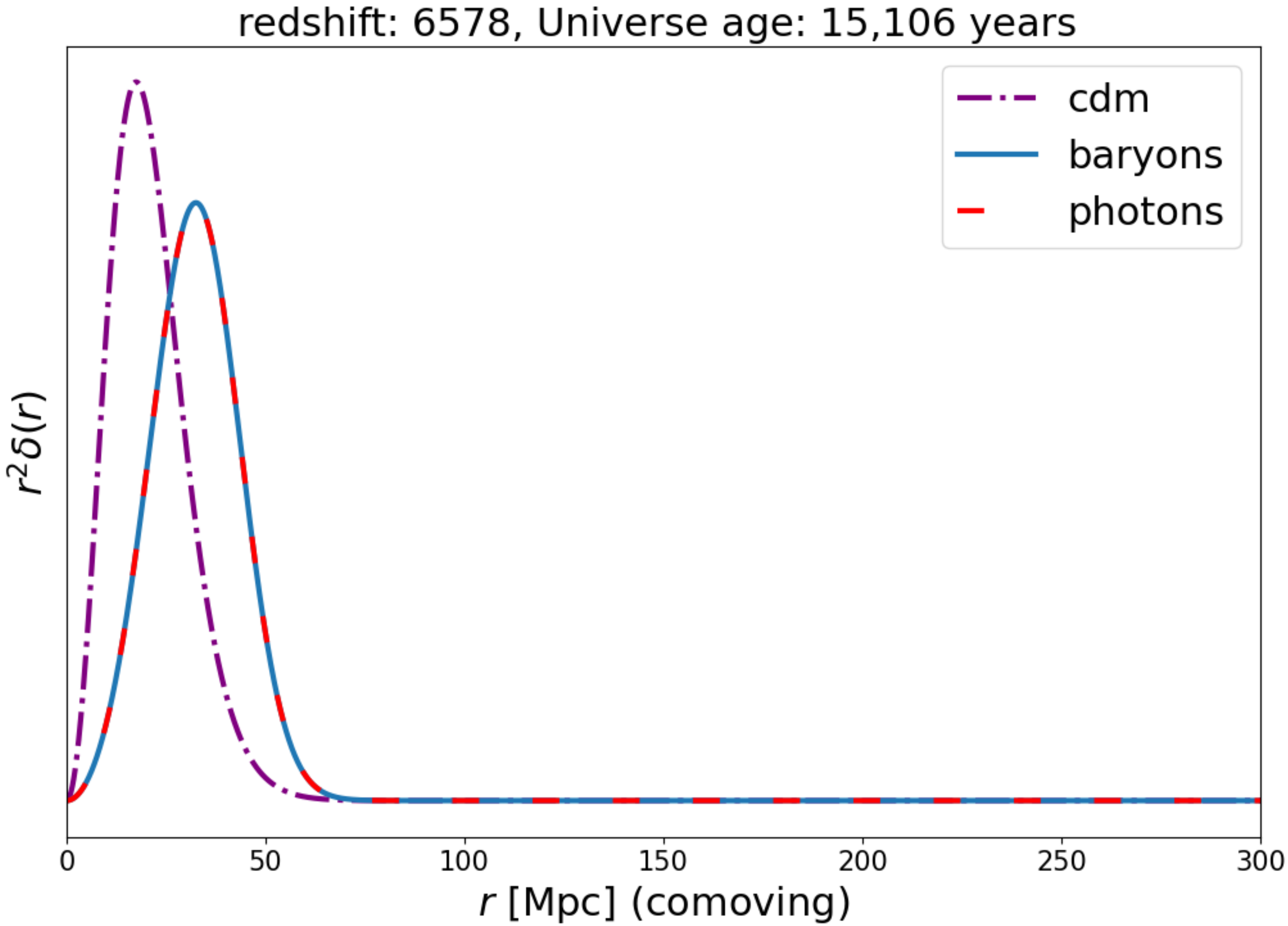

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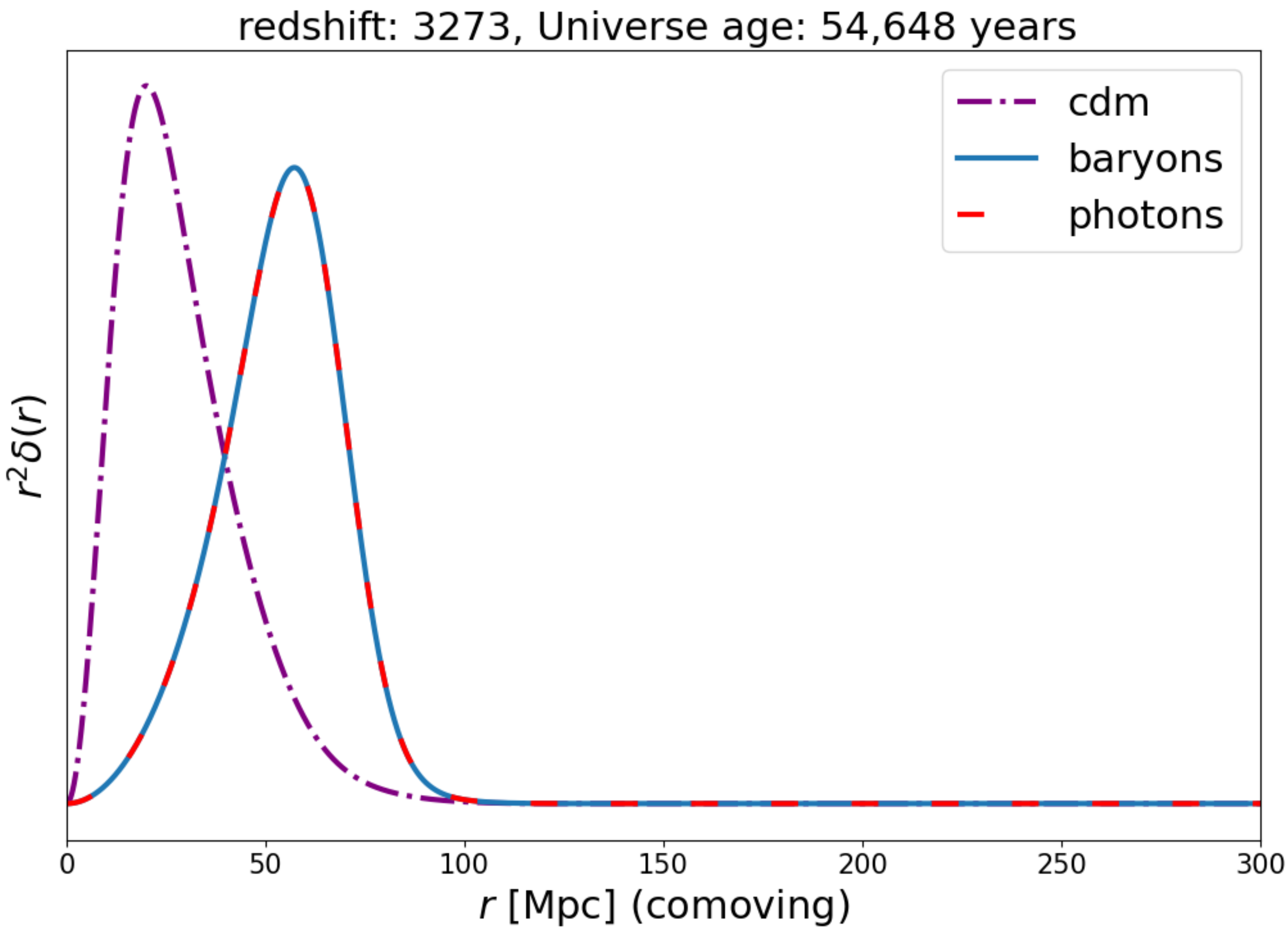

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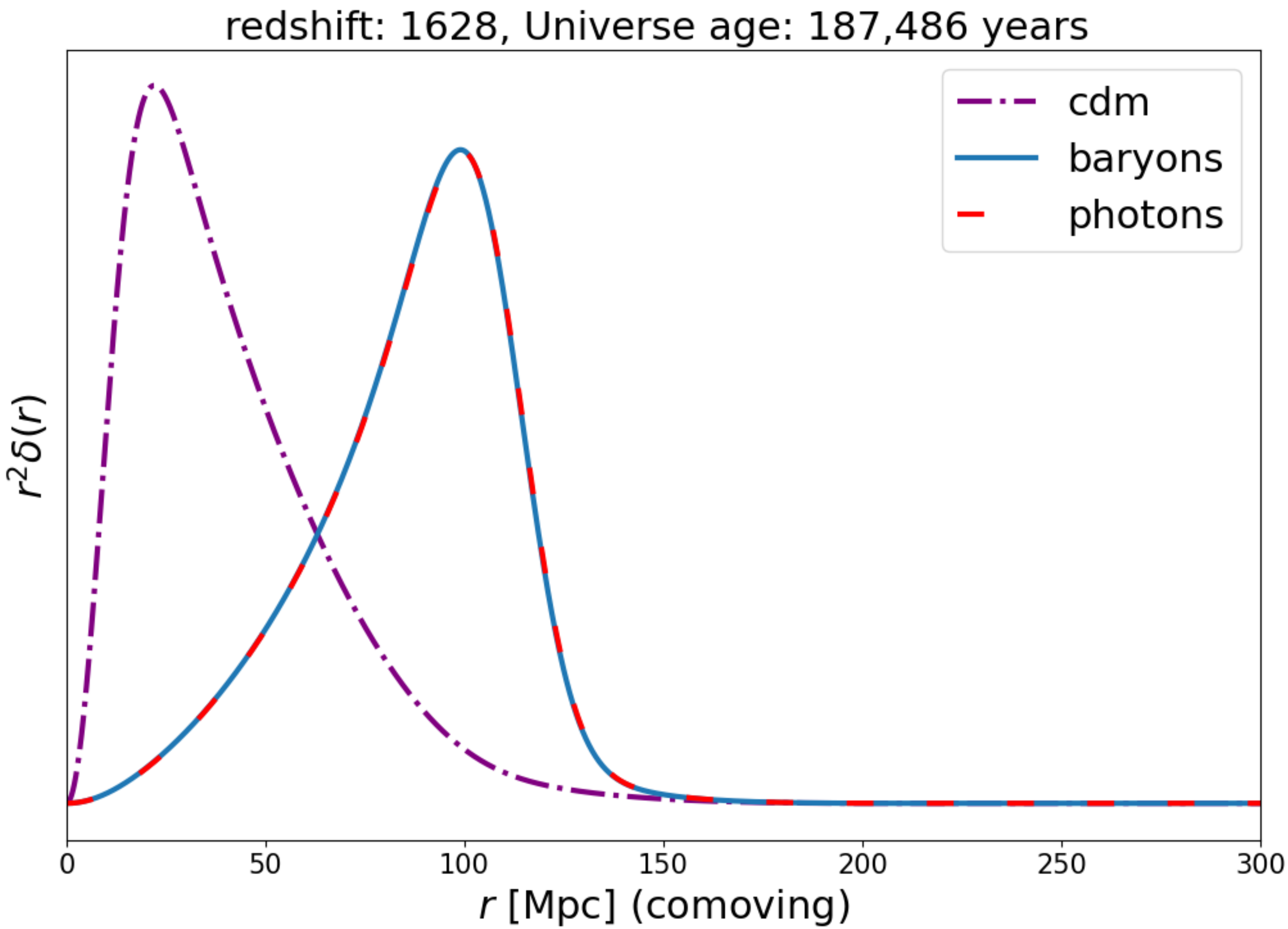

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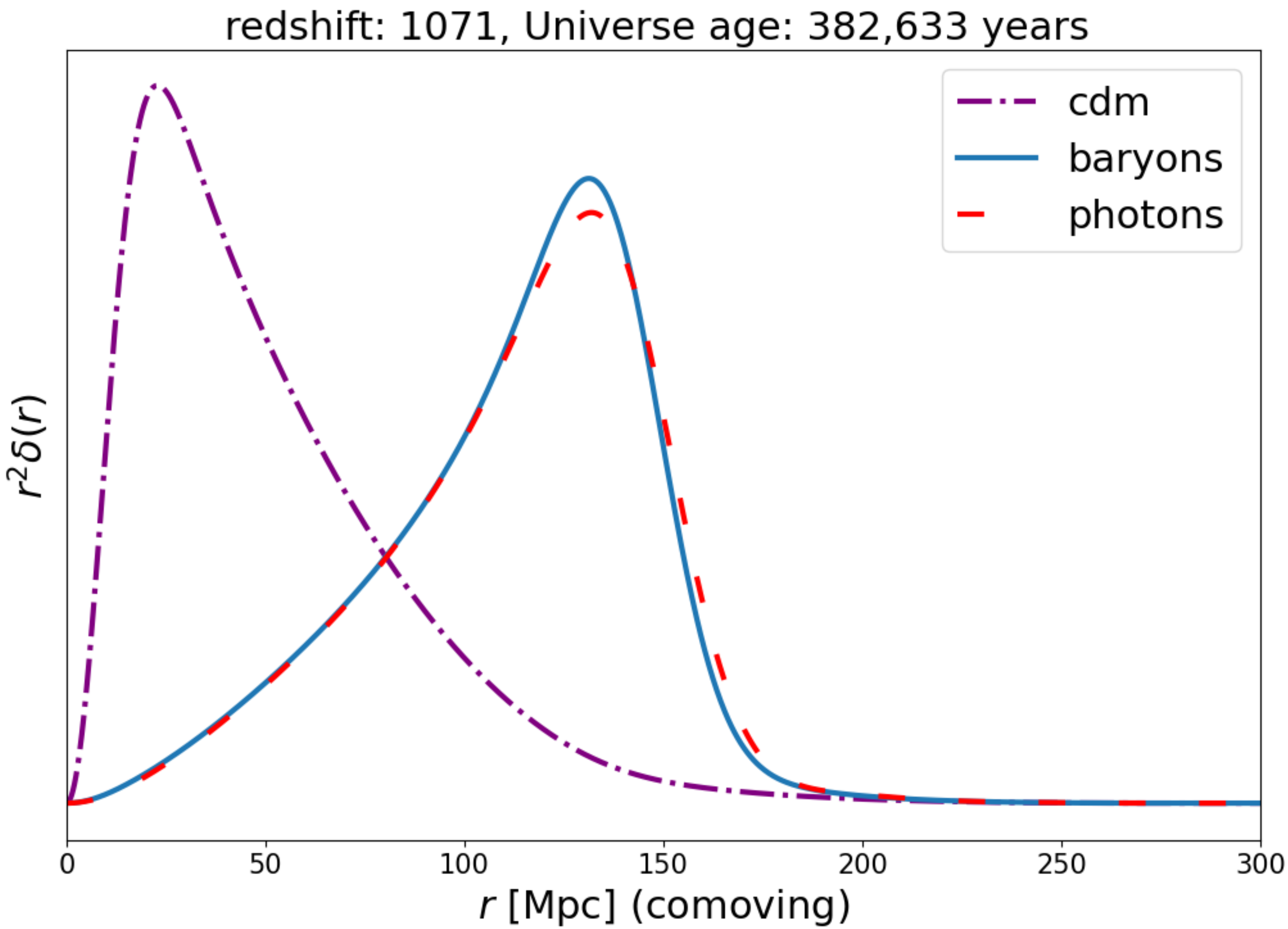

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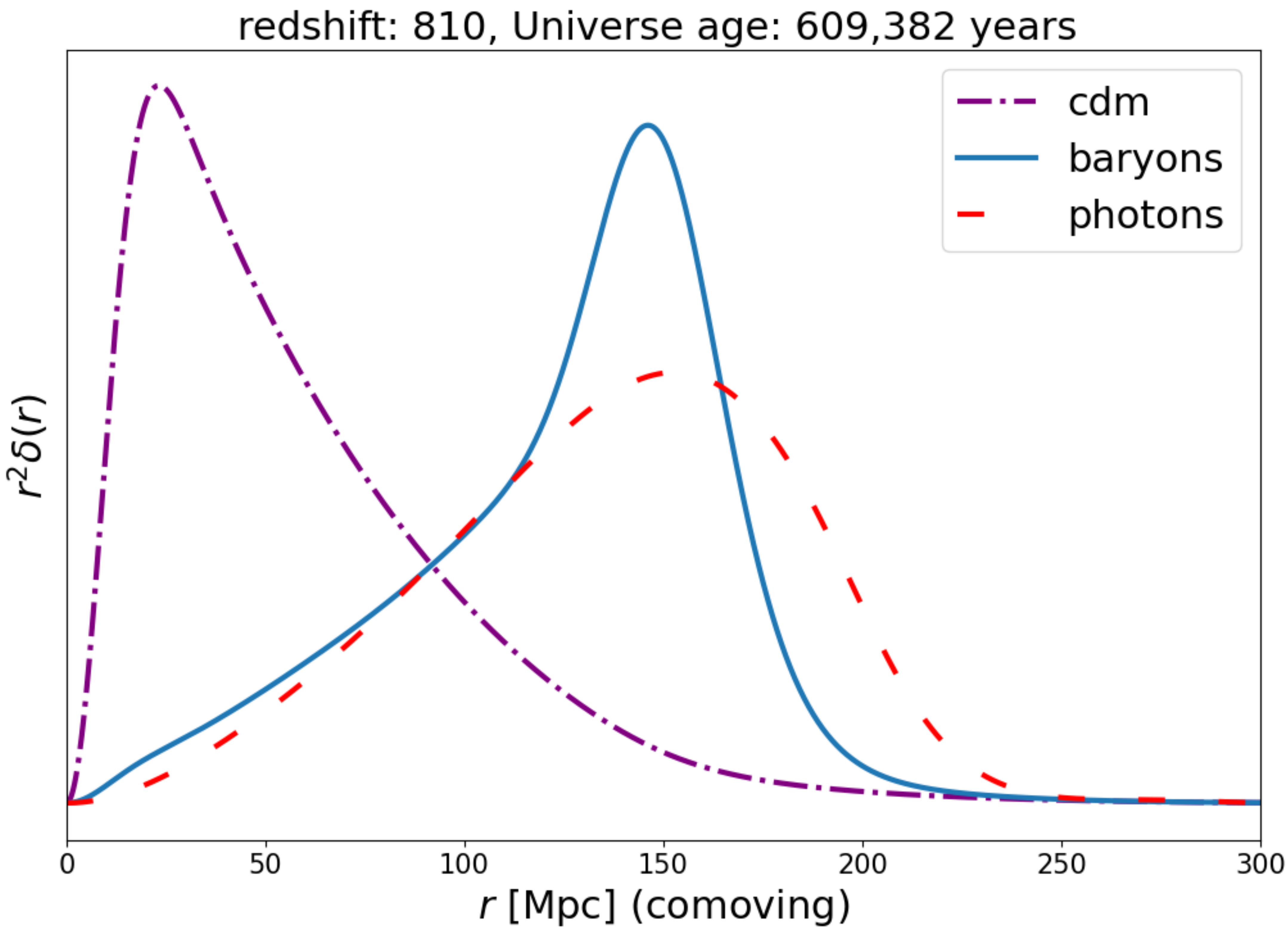

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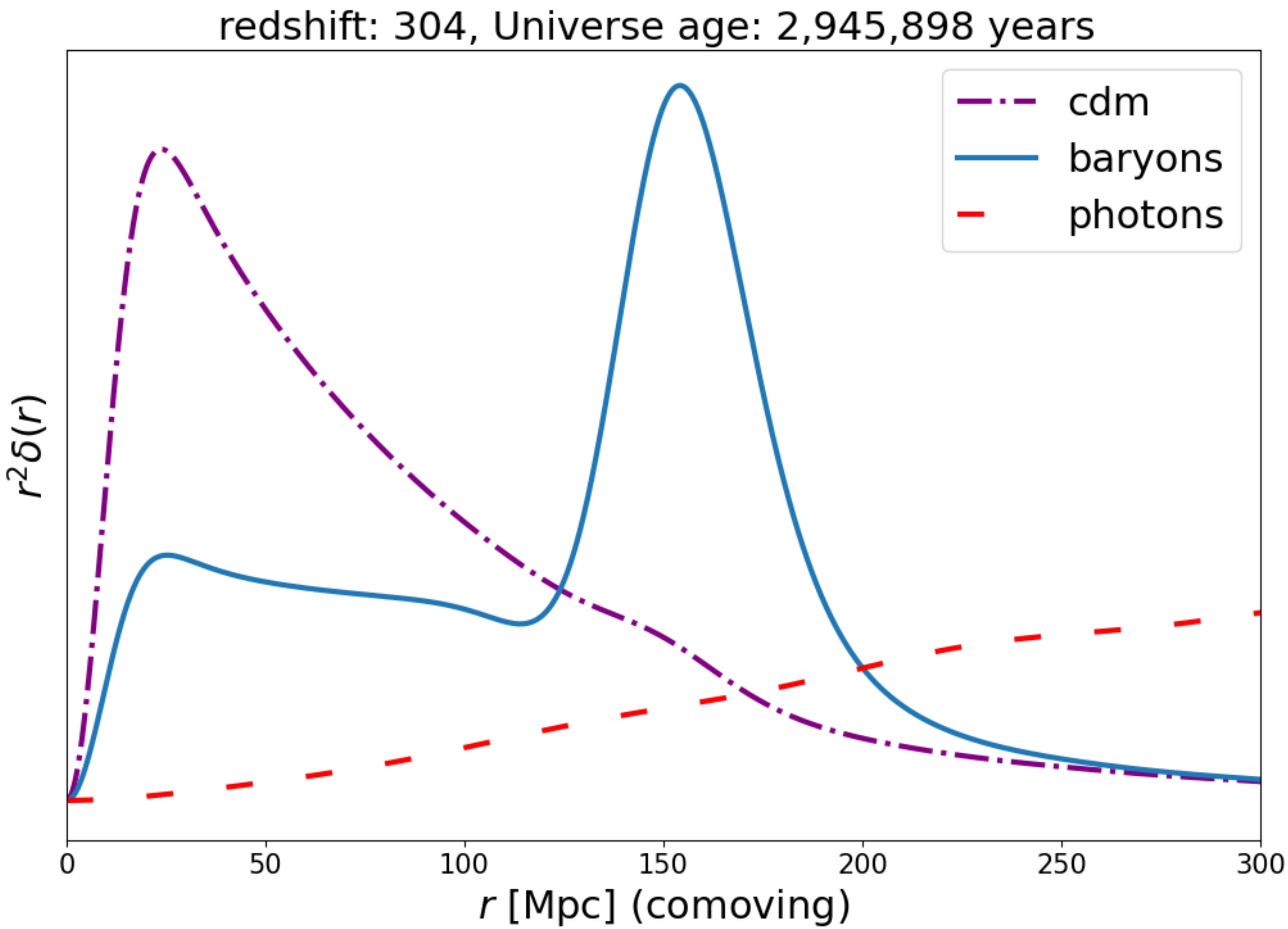

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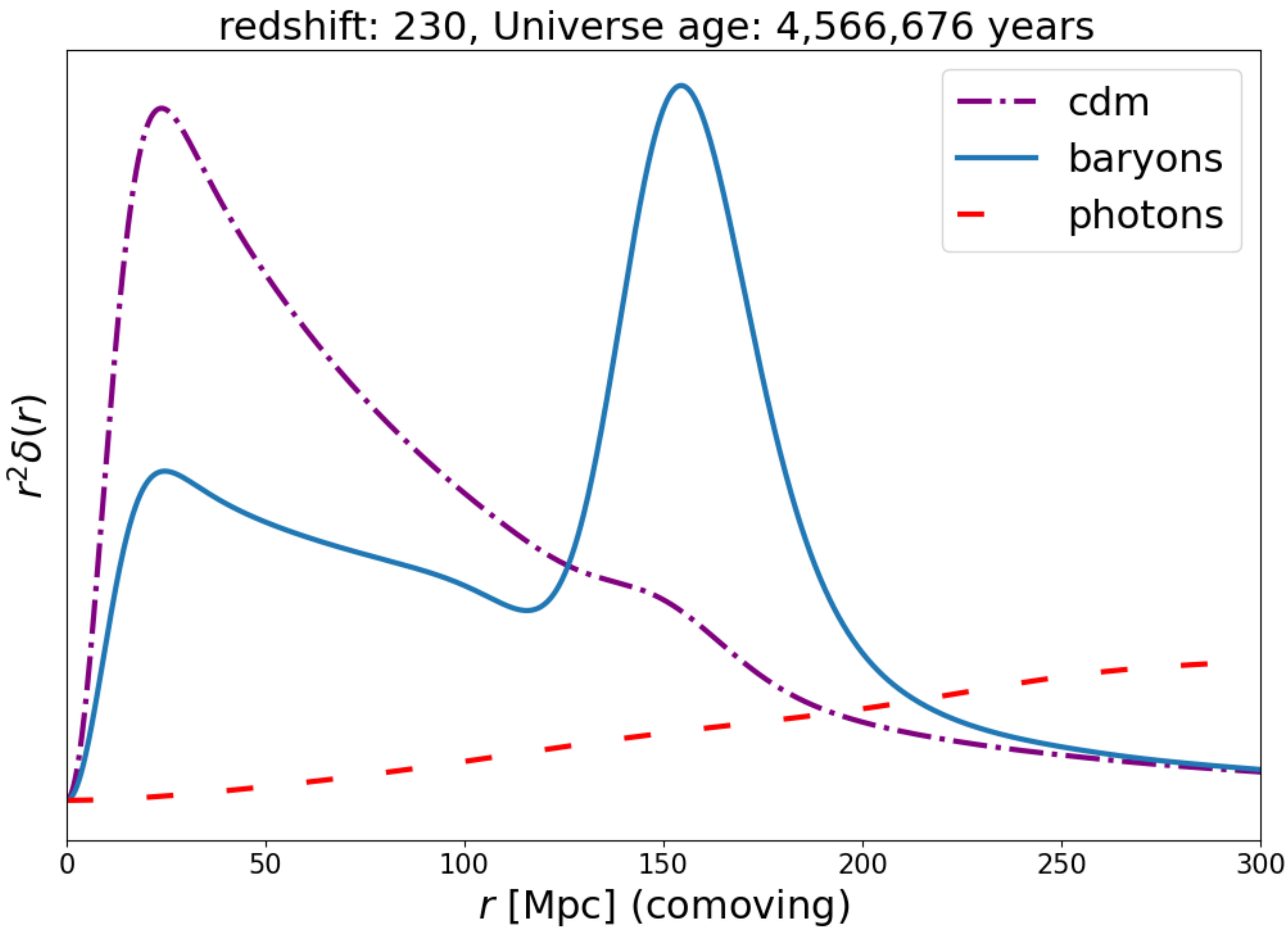

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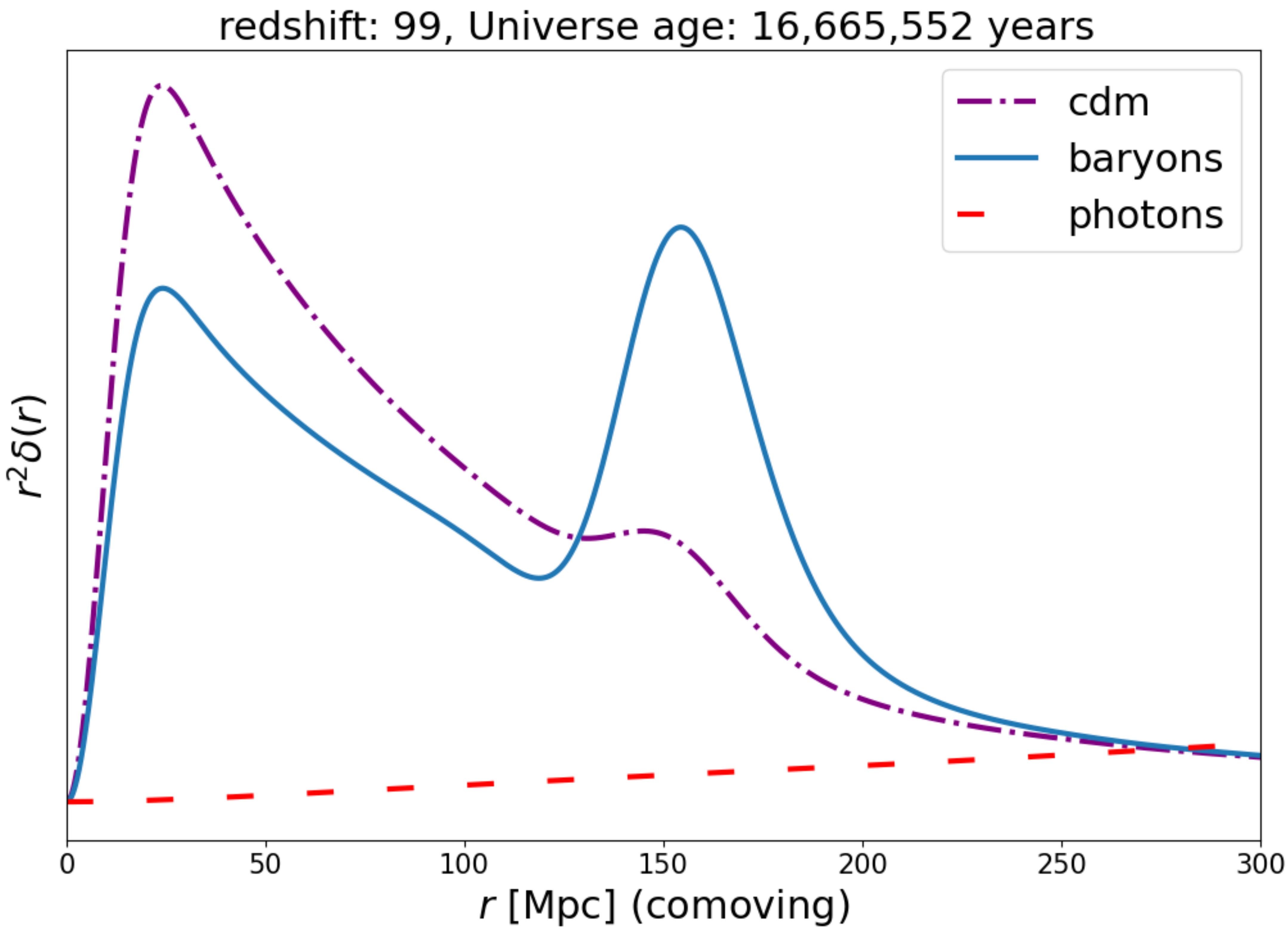

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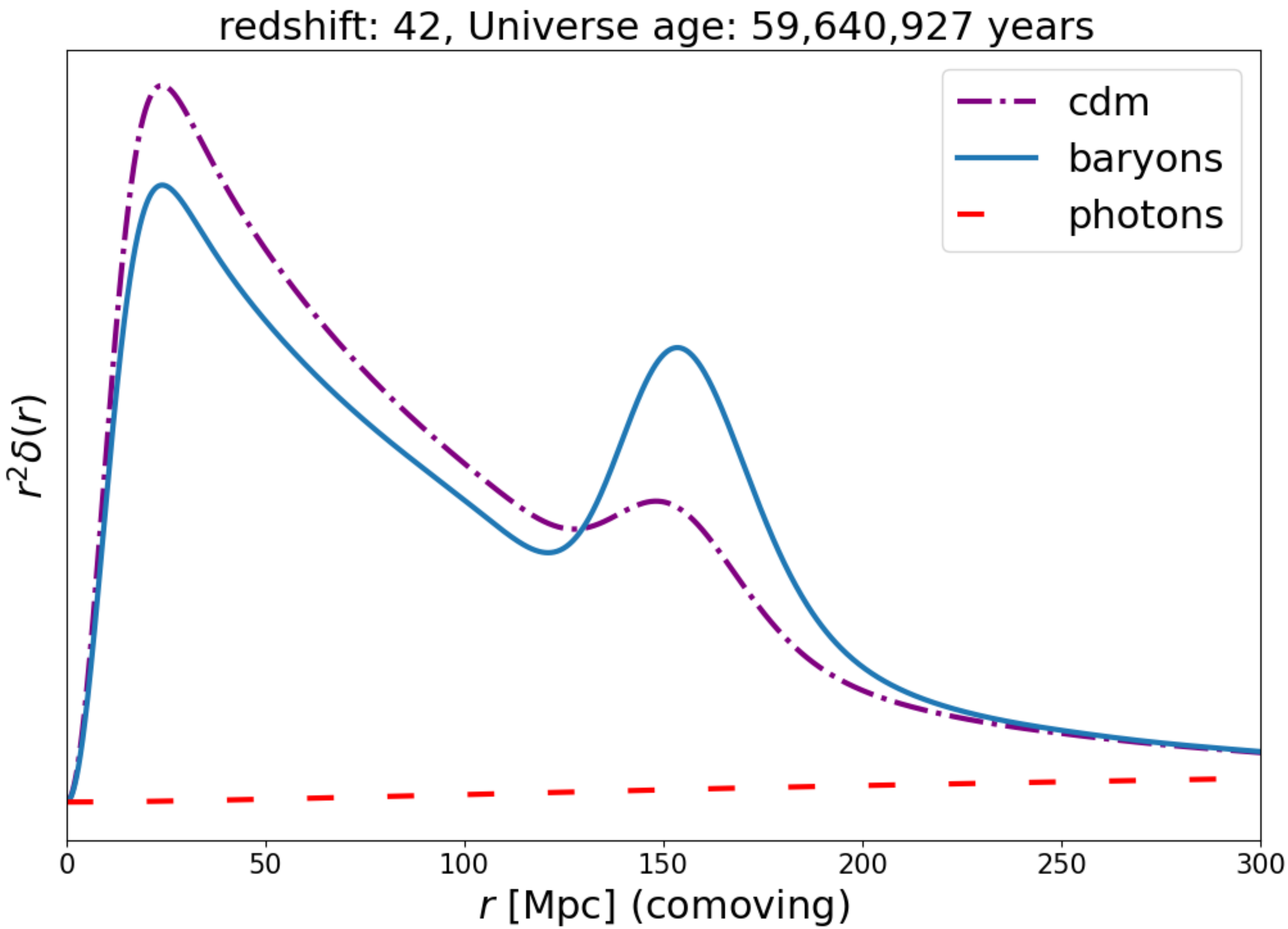

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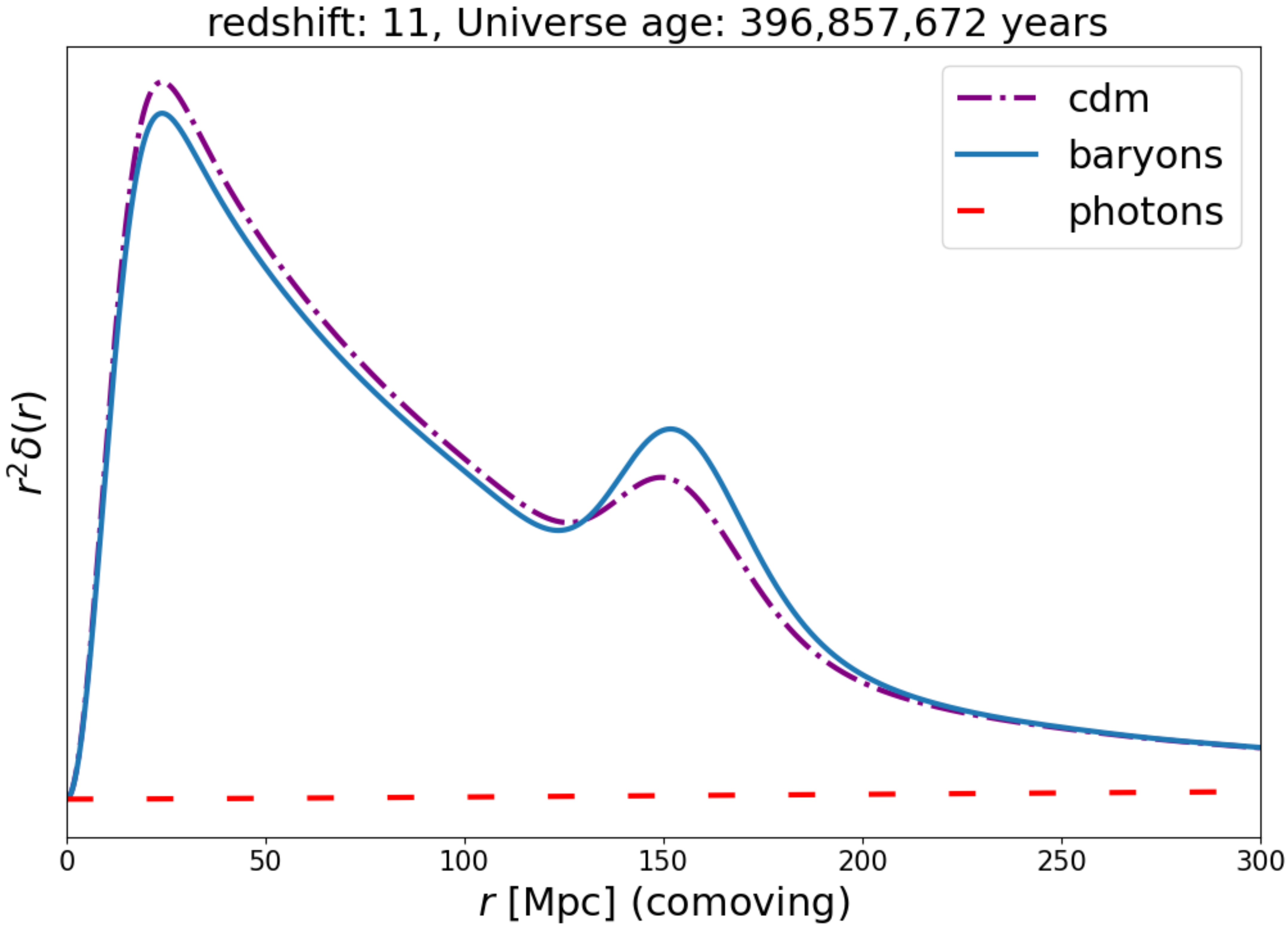

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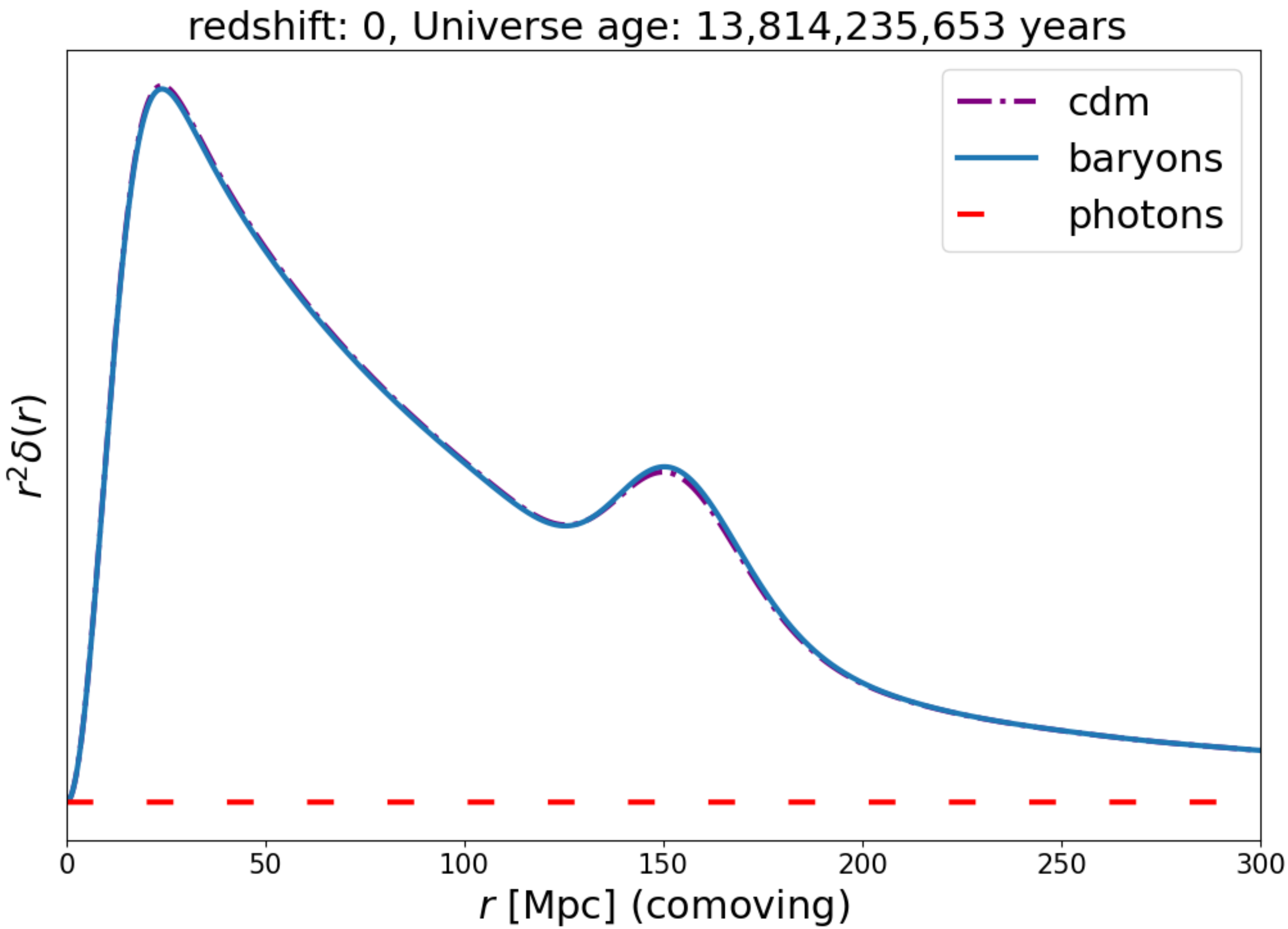

Of course the Universe is more complex than having just a single perturbation,
And to go further we need a theory of initial perturbation.

In the standard model of cosmology, this theory is inflation

In the inflation paradigm, primordial perturbations arise from quantum fluctuations
That are stretched out during a period of accelerated expansion of the Universe.

There is no deterministic description of quantum fluctuations, to describe
The distribution of initial perturbation in the Universe we need to use
statistical methods.

One word on statistical field theory

We are familiar to the standard gaussian distribution for a single variable (for example the gaussian distribution for the variable a, of mean mu and standard deviation sigma)

$$P(a) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{1}{2}\left(\frac{a-\mu}{\sigma}\right)^2$$

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Let's generalize this formula for a set of independent gaussian variables

$$P(\boldsymbol{a}) = \frac{1}{(2\pi)^{N/2}\sigma^N} \exp -\frac{1}{2}\sum_{i=1}^{N}\left(\frac{a_i-\mu}{\sigma}\right)^2$$

$\boldsymbol{a}$ is now a vector of N random variables

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We can generalize this formula further, for a set of correlated gaussian variables

$$P(\boldsymbol{a}) = \frac{1}{(2\pi)^{N/2}\det(\Sigma)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(a_i-\mu_i)\left(\Sigma^{-1}\right)_{ij}(a_j-\mu_j)\right]$$

One word on statistical field theory

$$P(\boldsymbol{a}) = \frac{1}{(2\pi)^{N/2}\det(\Sigma)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(a_i - \mu_i)\left(\Sigma^{-1}\right)_{ij}(a_j - \mu_j)\right]$$

Σ_{ij} is called the covariance matrix of the random variable $\boldsymbol{a}$, it is defined as

$$\Sigma_{ij} = \langle (a_i - \mu_i)(a_j - \mu_j) \rangle$$

And is used to assess how correlated the different component of $\boldsymbol{a}$ are

Together with the mean, it contains all information on the statistical properties of $\boldsymbol{a}$

One word on statistical field theory

In cosmology, at the end of inflation, we expect perturbations to follow a multivariate Gaussian distribution, for example the cold dark matter density:

$$P(\boldsymbol{\delta_{\mathrm{cdm}}}) = \frac{1}{(2\pi)^{N/2}\det(\Sigma)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\delta_{\mathrm{cdm,i}}\right)\left(\Sigma^{-1}\right)_{ij}\left(\delta_{\mathrm{cdm,j}}\right)\right]$$

Where $\delta_{\mathrm{cdm,i}} = \delta_{\mathrm{cdm}}(\boldsymbol{x}_i)$

So the covariance matrix associated to the dark matter density field after inflation will contain all information on the statistical properties of the field

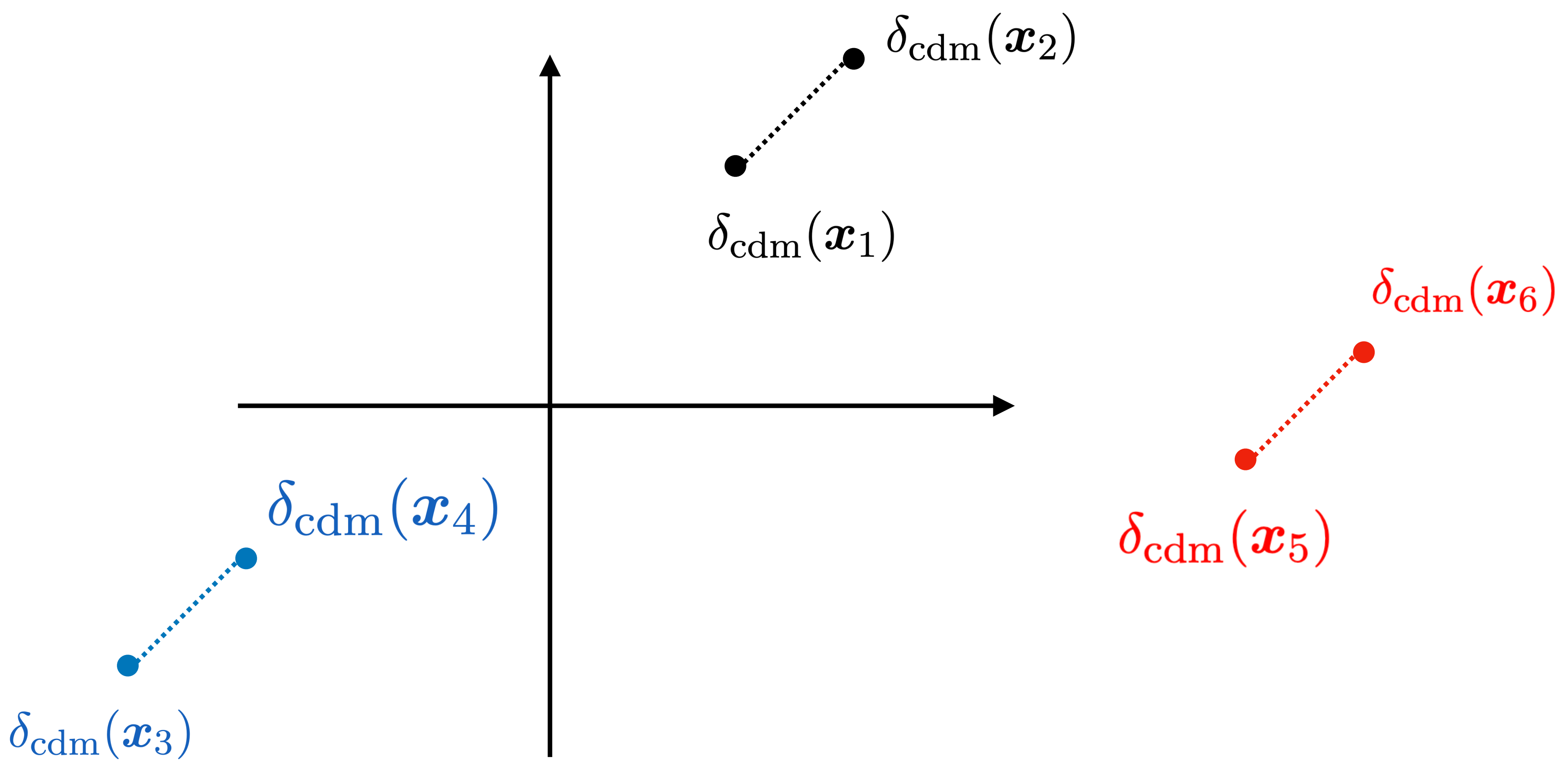
$$\Sigma_{ij} = \langle \delta_{\mathrm{cdm}}(\boldsymbol{x}_i)\delta_{\mathrm{cdm}}(\boldsymbol{x}_j)\rangle$$

One word on statistical field theory

We make two simplifying assumptions, first we assume that the statistical properties of the field are homogeneous

$$\Sigma_{ij} = \langle \delta_{\rm cdm}(\boldsymbol{x}_i)\delta_{\rm cdm}(\boldsymbol{x}_j)\rangle = f(\boldsymbol{x}_i - \boldsymbol{x}_j)$$

(there is no preferred location in the Universe)



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$$\Sigma_{ij} = \langle \delta_{\rm cdm}(\boldsymbol{x}_i)\delta_{\rm cdm}(\boldsymbol{x}_j)\rangle = f(|\boldsymbol{x}_i - \boldsymbol{x}_j|) = f(r)$$

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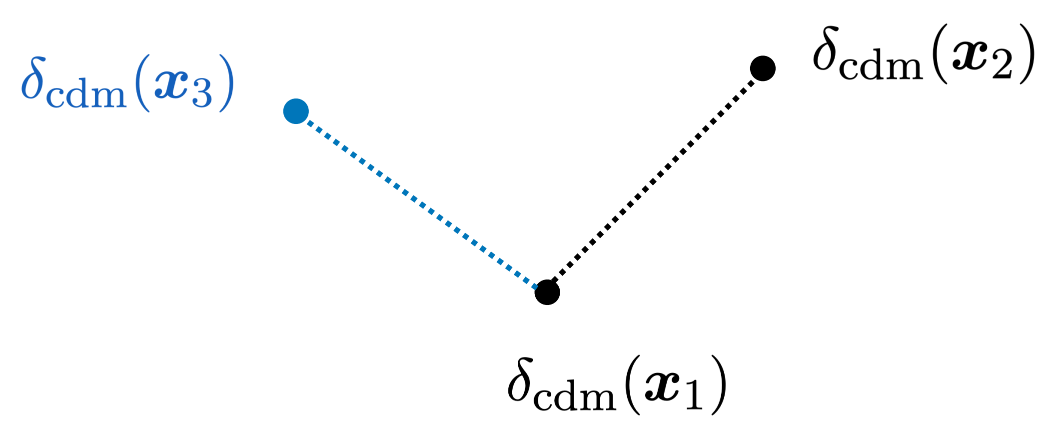

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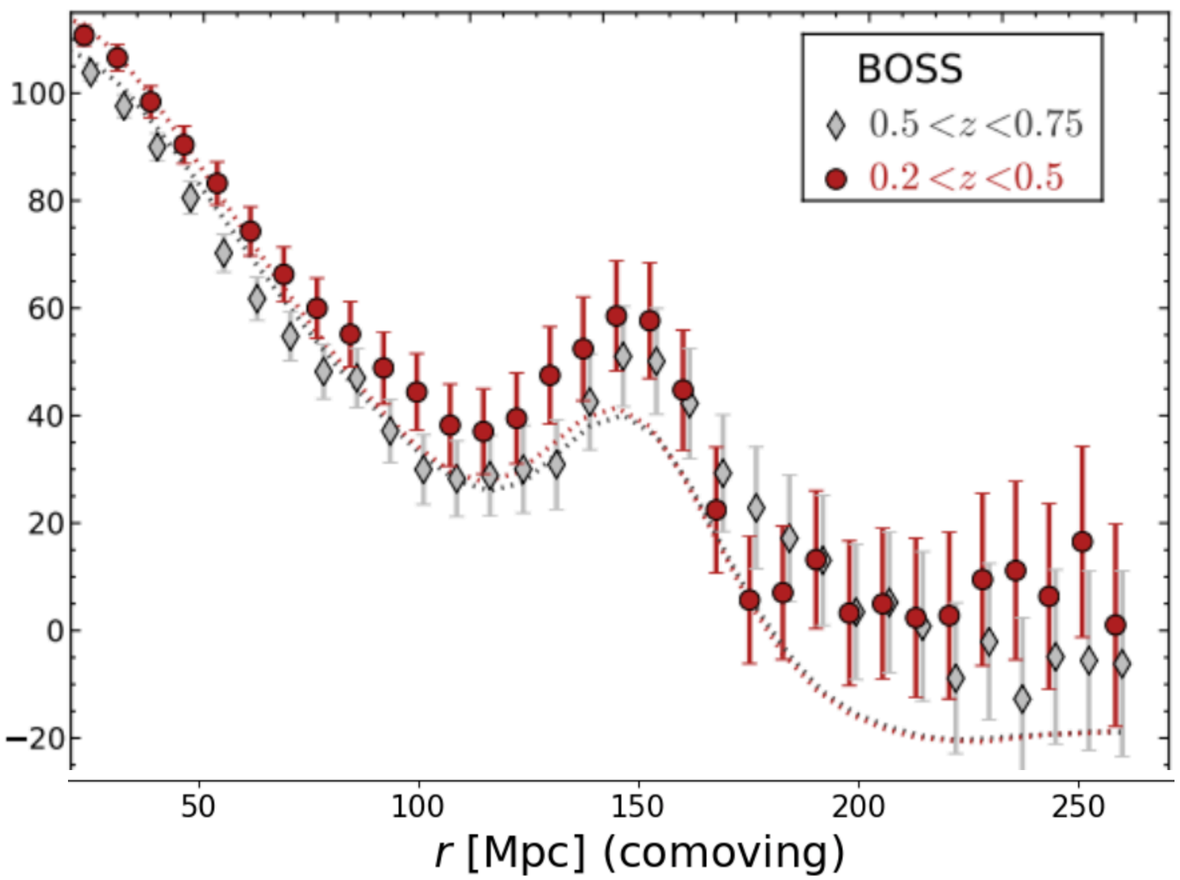
This allows us to describe the statistical properties of the field, after inflation With a single one dimensional function $f(r)$

This one dimensional function is called the correlation function and we usually denote it by $\xi(r)$

One word on statistical field theory



Correlation functions are the main observables in cosmology, since
we can predict them directly from theory

One word on statistical field theory

Alternatively, since we usually solve equation in Fourier space for simplicity reason we can describe the statistical properties of the field by its power spectrum.

The power spectrum is just the Fourier transform of the correlation function

$$\begin{aligned}
\xi(\boldsymbol{r}) &= \int d^3\boldsymbol{k} P(\boldsymbol{k}) e^{i\boldsymbol{k}.\boldsymbol{r}} \\
\xi(r) &= 4\pi \int dk P(k) j_0(kr) k^2 dk
\end{aligned}$$

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$$\xi(r) = \langle \delta(\boldsymbol{x}_i)\delta(\boldsymbol{x}_i + \boldsymbol{r})\rangle$$

$$P(k) = \langle \delta(\boldsymbol{k})\delta^*(\boldsymbol{k})\rangle$$

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In the simplest inflation theory, the statistical distribution of initial perturbations
Is described by a gaussian probability distribution function with the following power spectrum

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3} A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

Remember our equation system

$$
\begin{aligned}
\delta_c' + iku_c &= -3\phi' \\
u_c' + \frac{a'}{a}u_c &= -ik\ \psi \\
\delta_b' + iku_b &= -3\phi' \\
u_b' + \frac{a'}{a}u_b &= -ik\ \psi + \frac{\tau'}{R}[u_b + 3i\Theta_1]
\end{aligned}
$$

$$
k^2\phi + 3\frac{a'}{a}\left(\phi' - \ \psi\frac{a'}{a}\right) = 4\pi G a^2 \delta\rho
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$$
k^2(\phi + \psi) \sim 0
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Notice that we can solve the full system of equation for each Fourier mode independently.

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Notice that we can solve the full system of equation for each Fourier mode independently.

The solution for the differential equation for each species can therefore be written

$$
\delta_X(k,t) = T(k,t)\delta_X(k,t_{\text{ini}})
$$

Where the function $T_X(k,z)$ are called the transfer function

To compute the matter power spectrum today, all you need is the initial power spectrum and the matter transfer function computed at z = 0

$$\begin{aligned}
P_m(k, z=0) &= \langle \delta_m(k, z=0)\delta_m^*(k, z=0)\rangle \\
&= T_m^2(k, z=0)\langle \delta_m(k, z_{\rm ini})\delta_m(k, z_{\rm ini})\rangle \\
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The primordial power spectrum is fully specified with two parameters

$$A_s, n_s$$

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3} A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

The measured values of these parameters Helps us constraining the inflation model

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$$

The transfer functions represents the solution to the system of coupled differential equations, it depends on the baryon, dark matter, and radiation energy density

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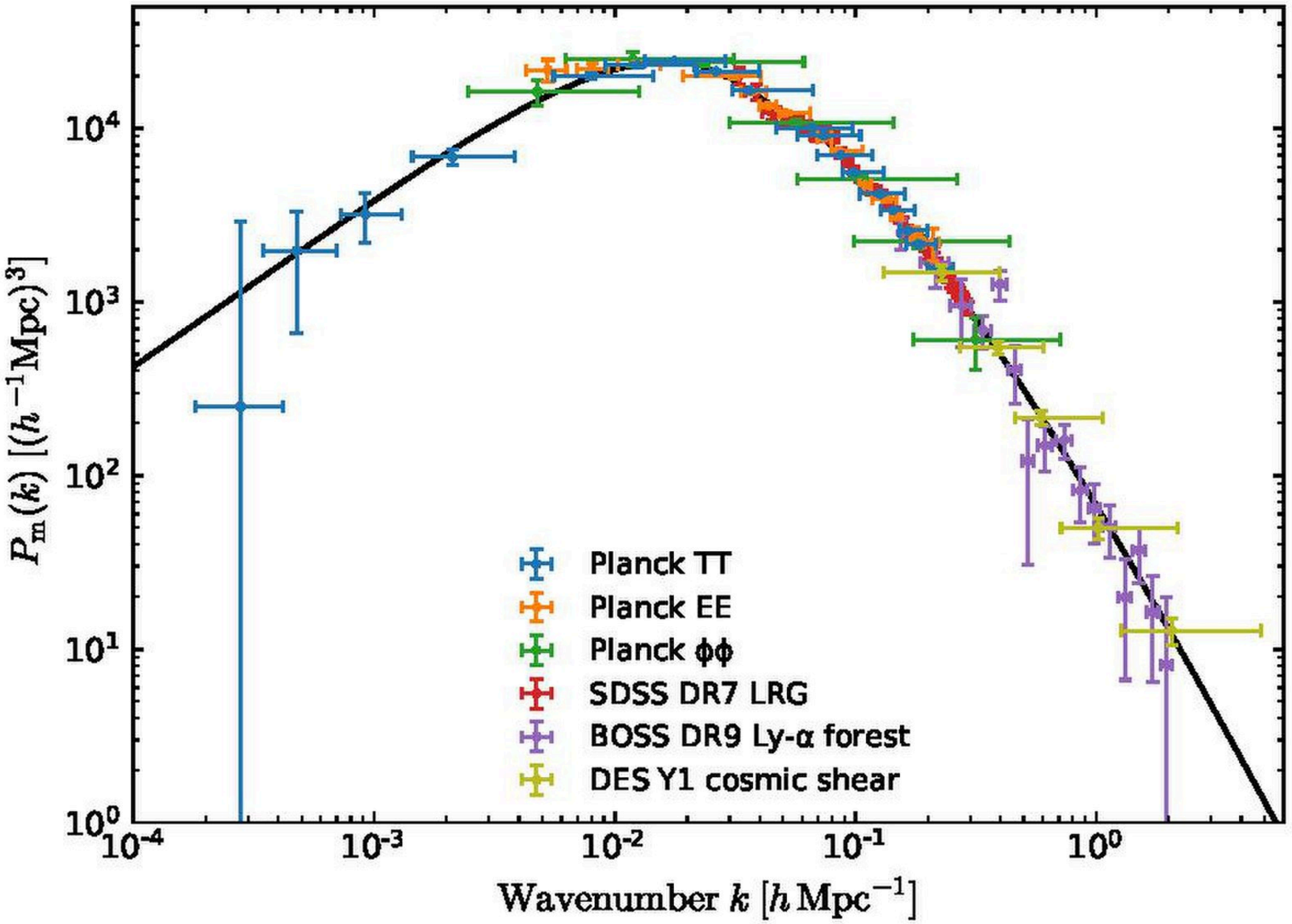

The matter power spectrum
is pretty well constrained by many
different observables, and is one of
The key foundation of our current
Cosmological model.

Solving our equation system as a function of time:

$$
\begin{aligned}
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u_c' + \frac{a'}{a}u_c &= -ik\ \psi \\
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\end{aligned}
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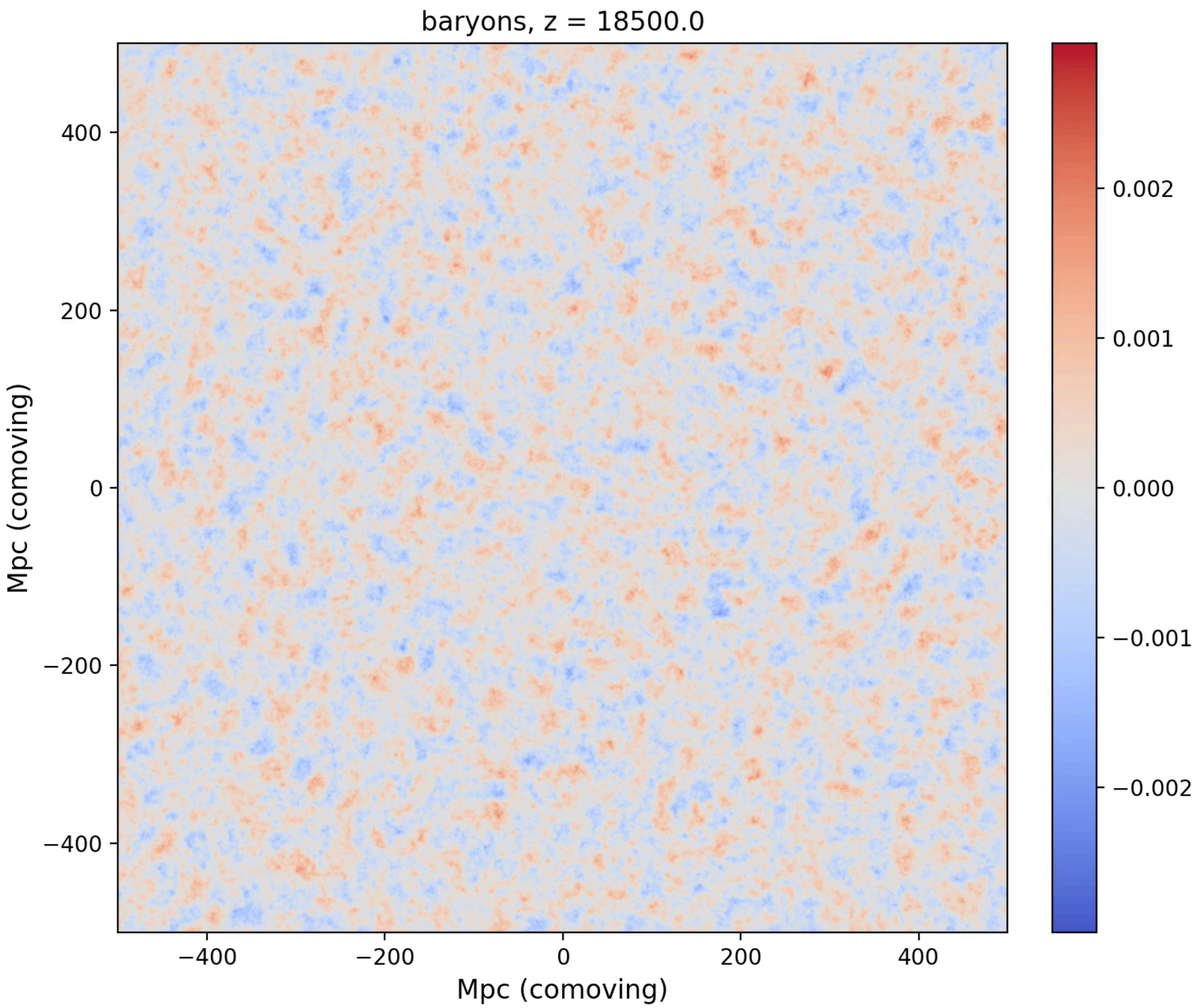
$$
k^2\phi + 3\frac{a'}{a}\left(\phi' - \psi\frac{a'}{a}\right) = 4\pi G a^2 \delta\rho
$$

$$
k^2(\phi + \psi) \sim 0
$$

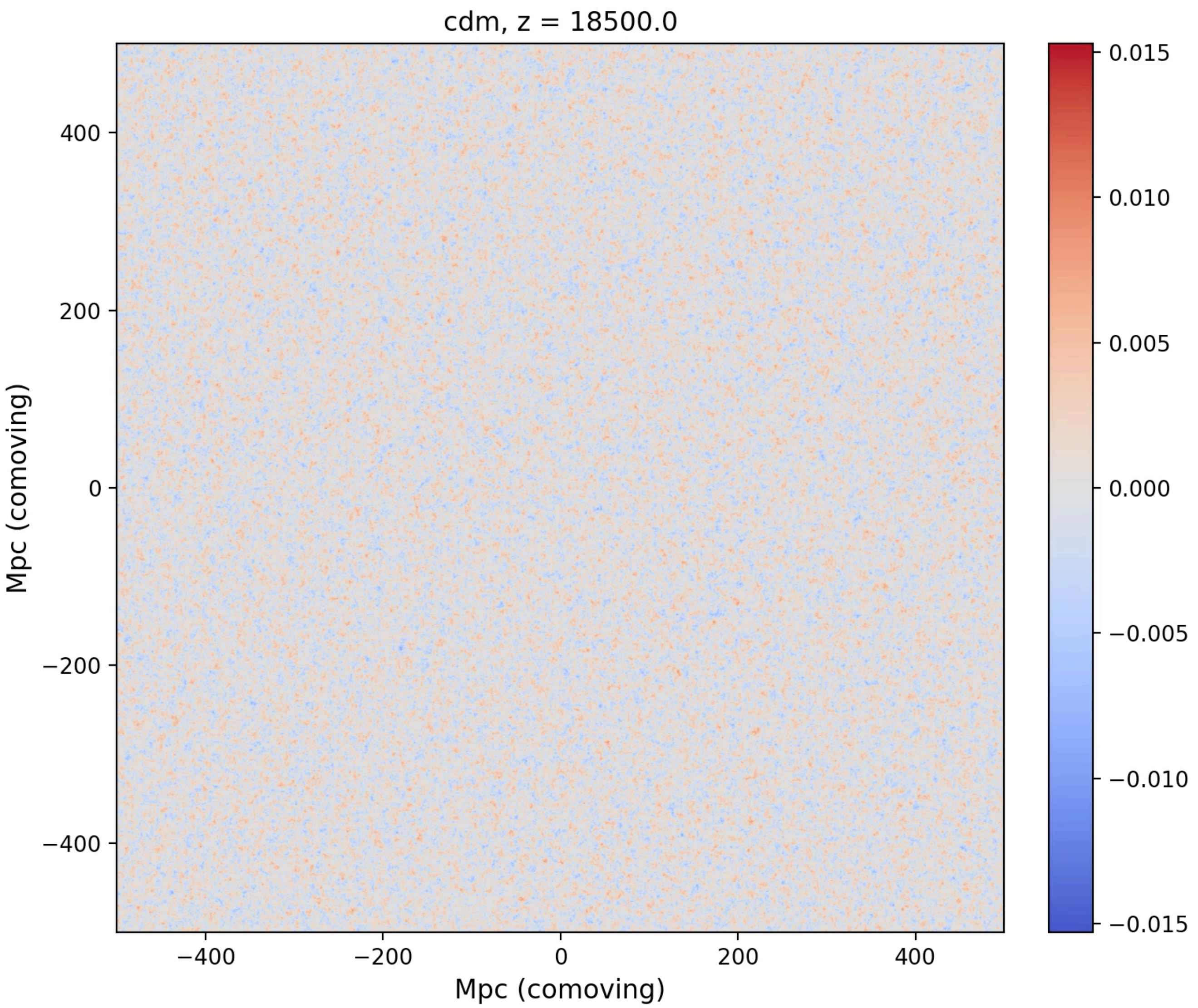
Allows us to compute the transfer functions at any redshifts.
This means we can simulate the evolution of perturbations as a function of time/redshift. This is useful to gain some intuition in particular for the role of the dark and baryonic matter.

LCDM

Baryons



Dark matter



pre-decoupling (z = 20000-1100)

LCDM

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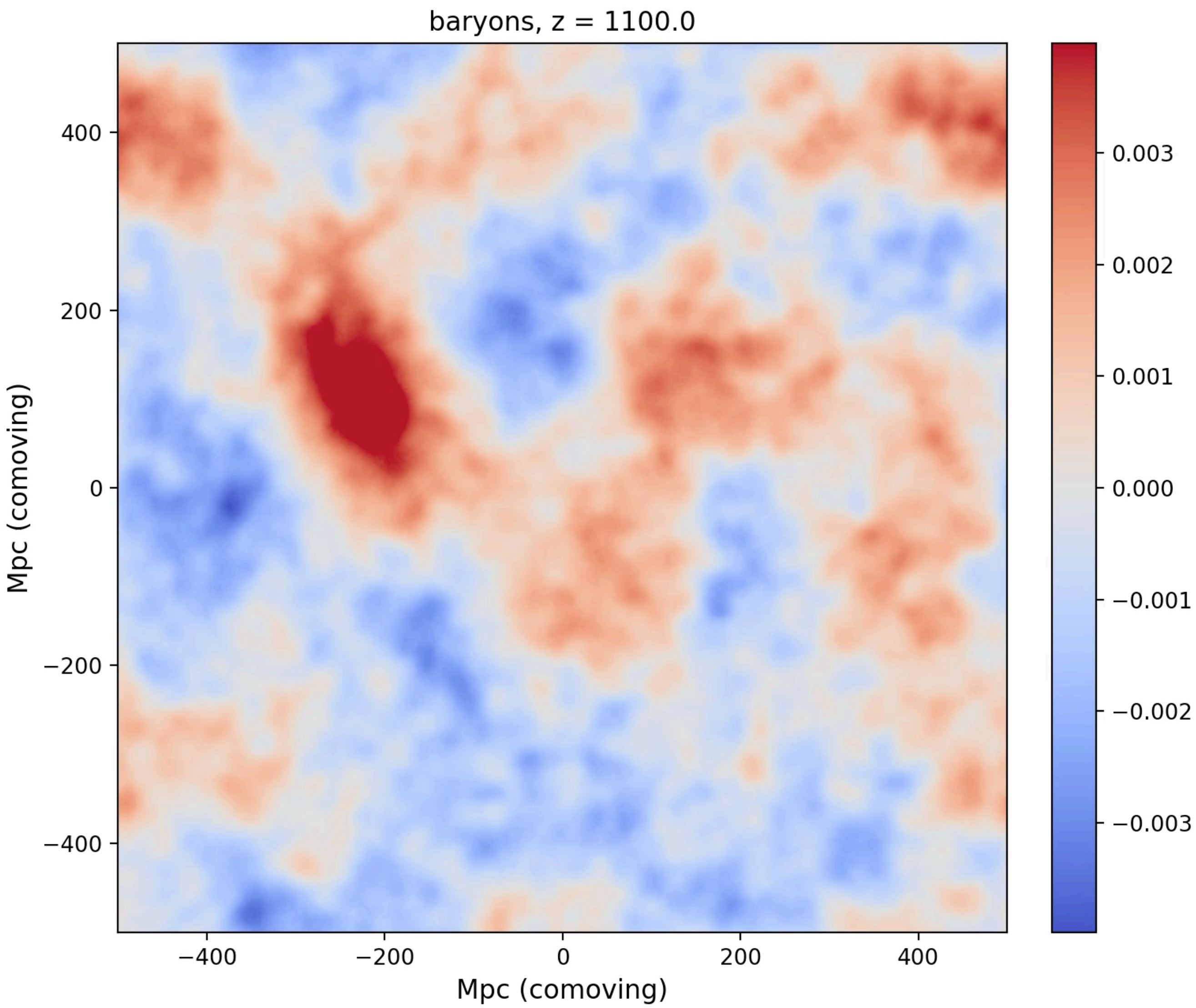

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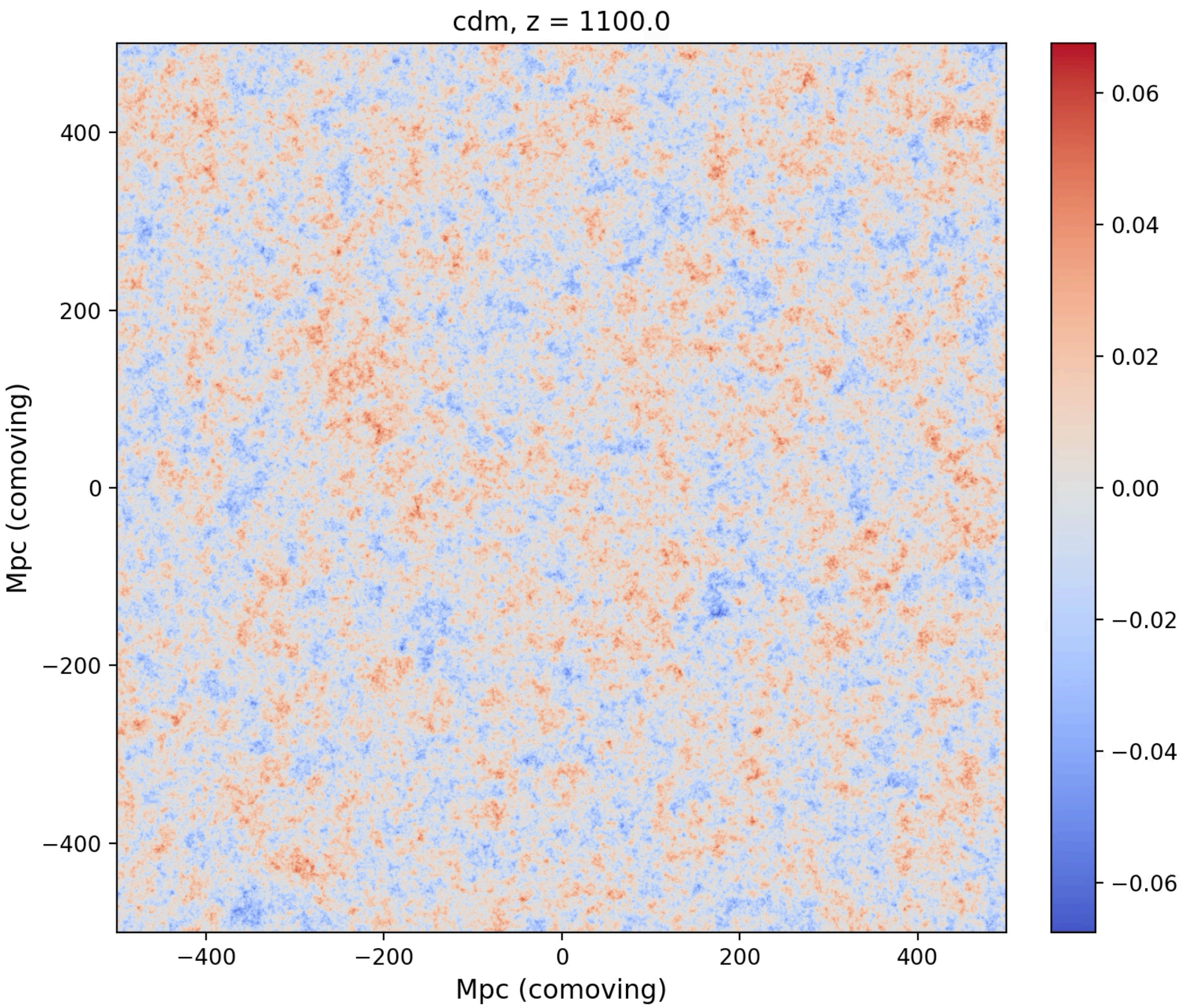
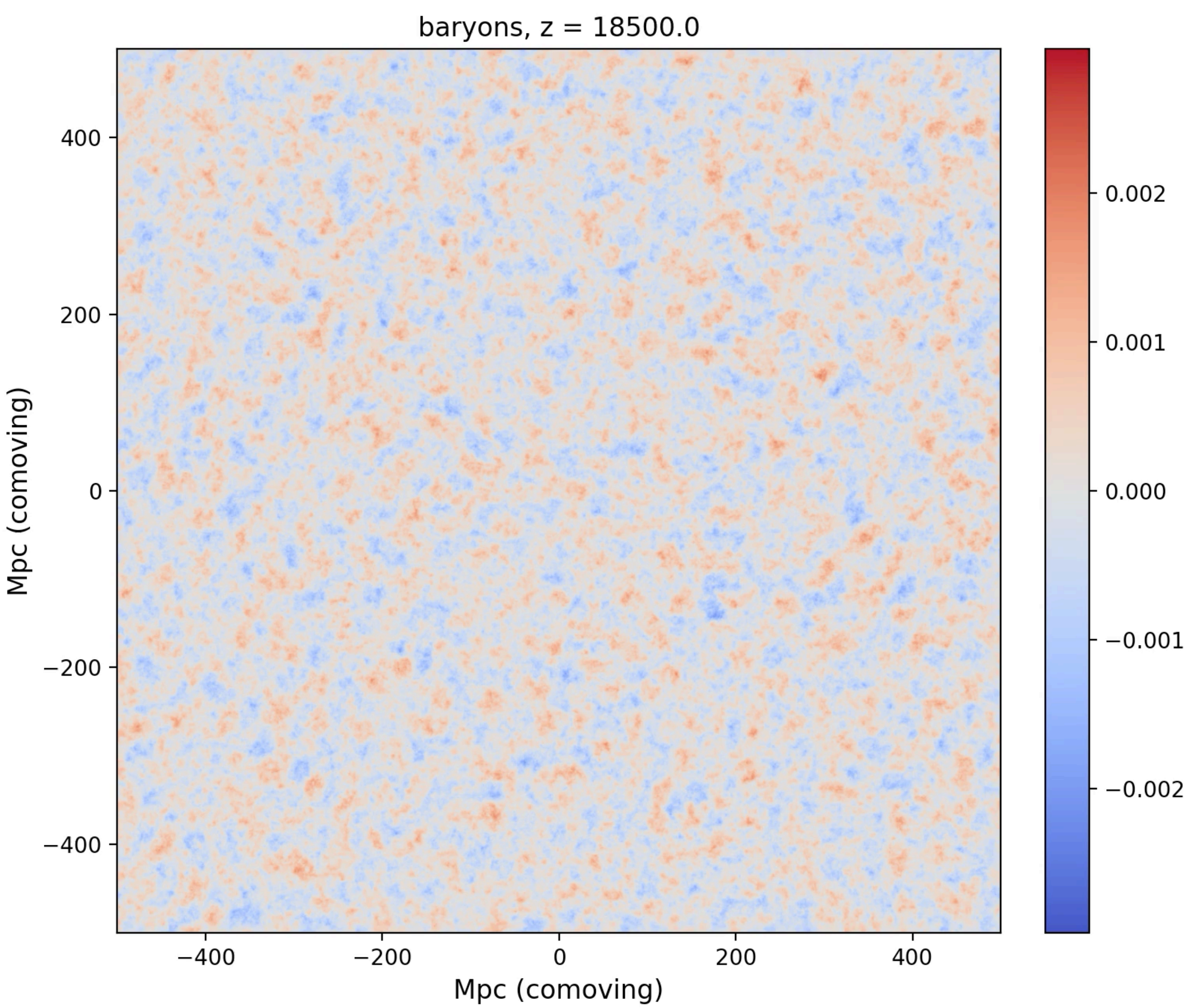
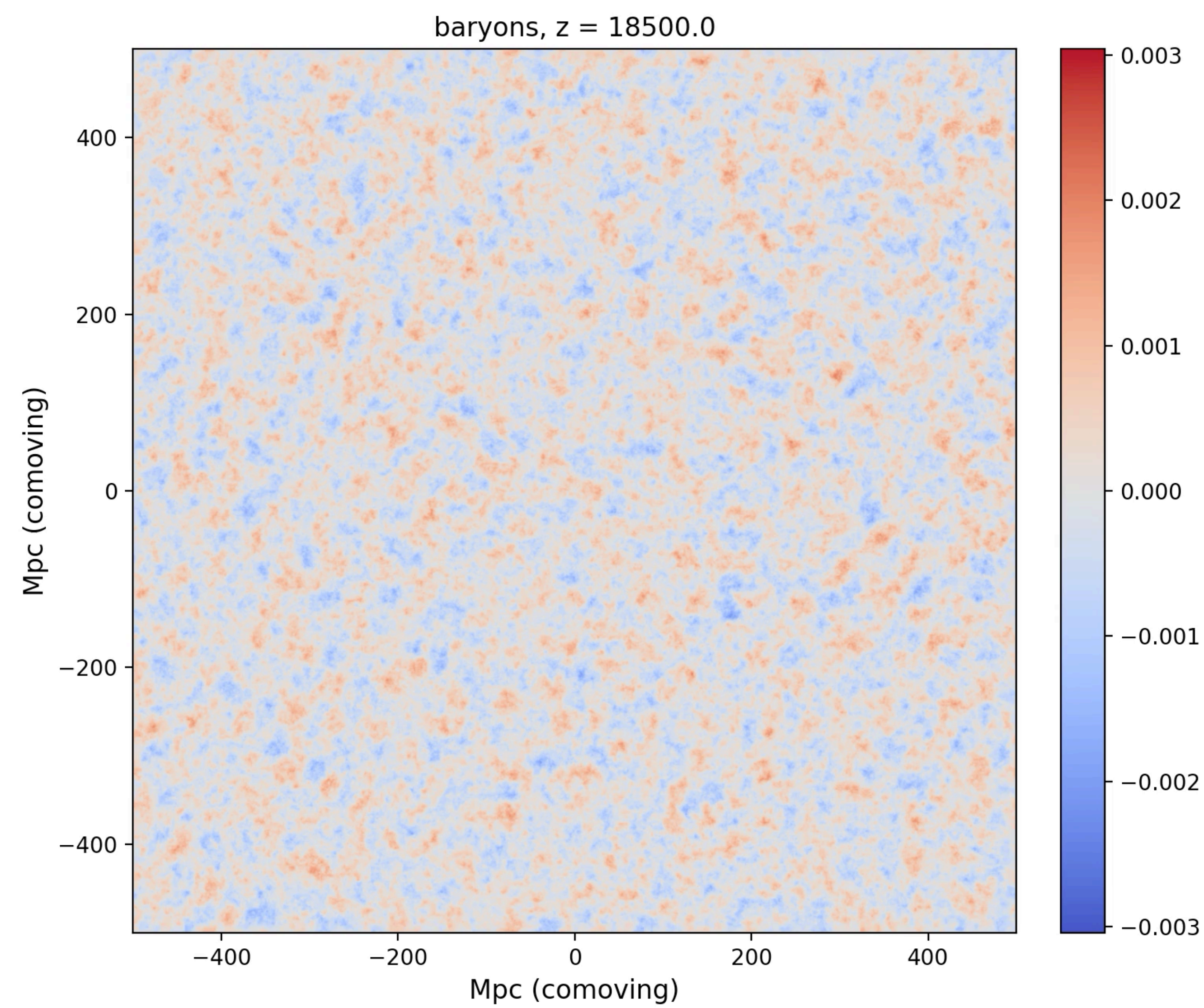


post-decoupling (z = 1100-100)

Baryons (LCDM)



Baryons (Universe with no dark matter)



pre-decoupling (z = 20000-1100)

Baryons (LCDM)

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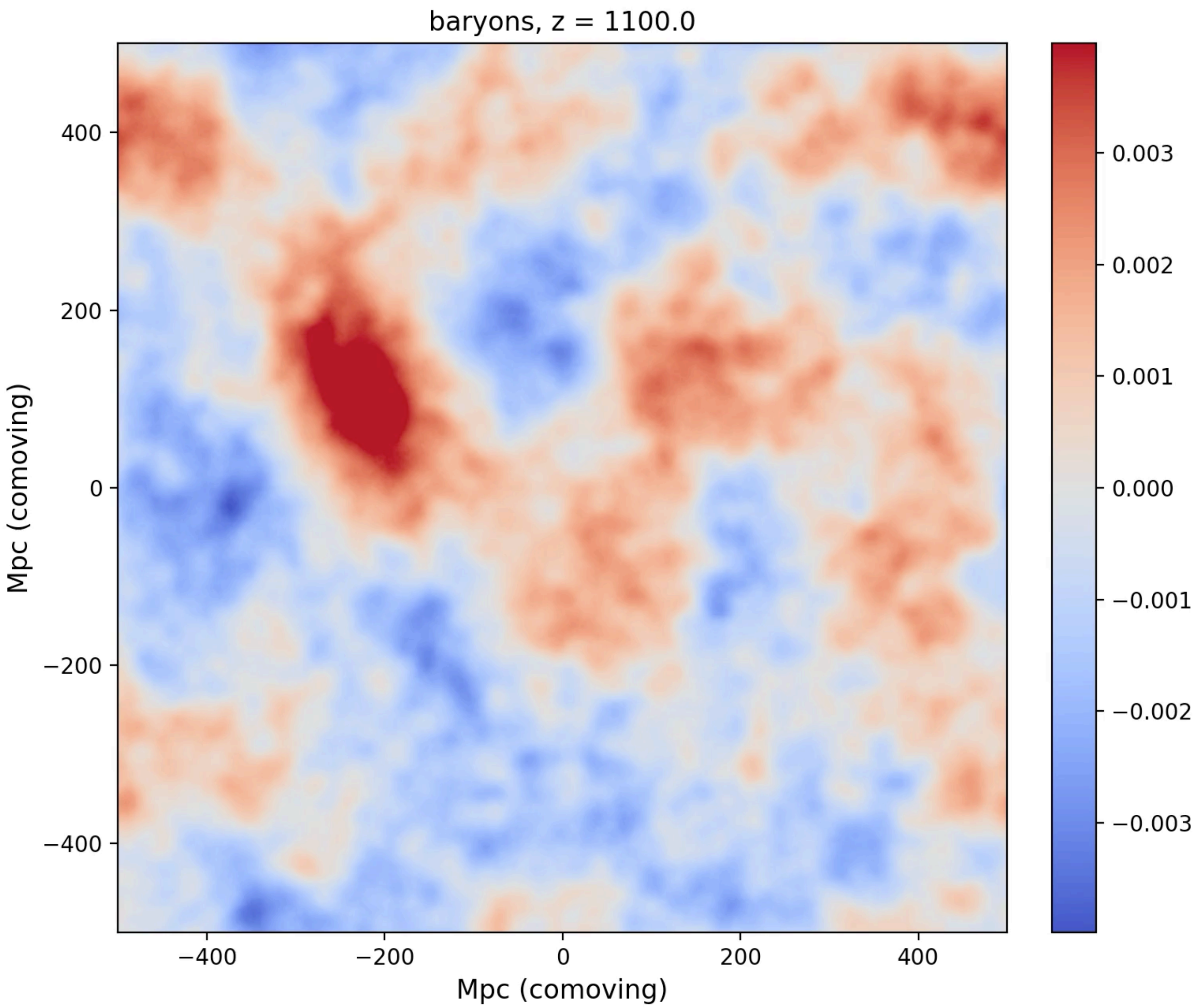
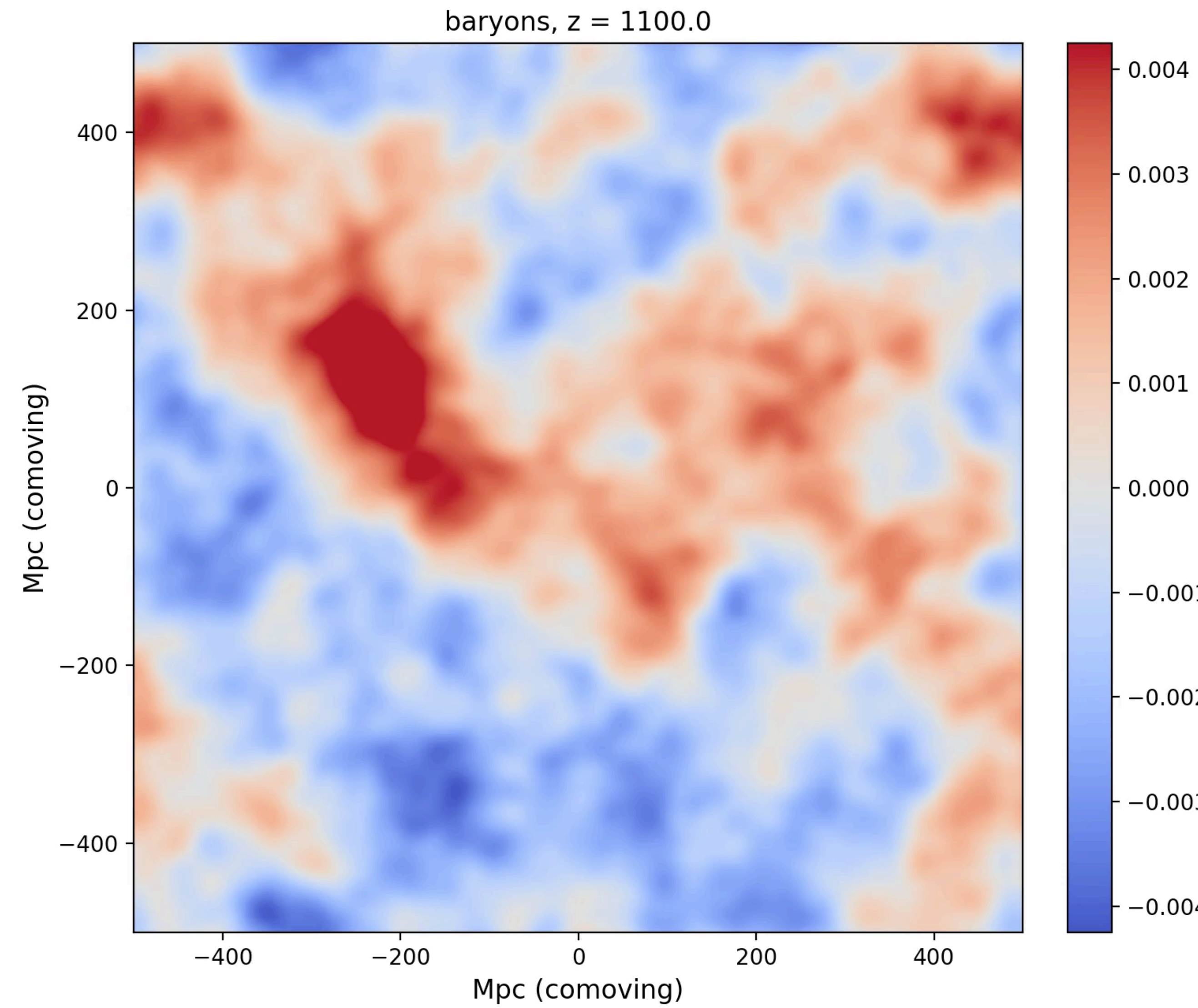


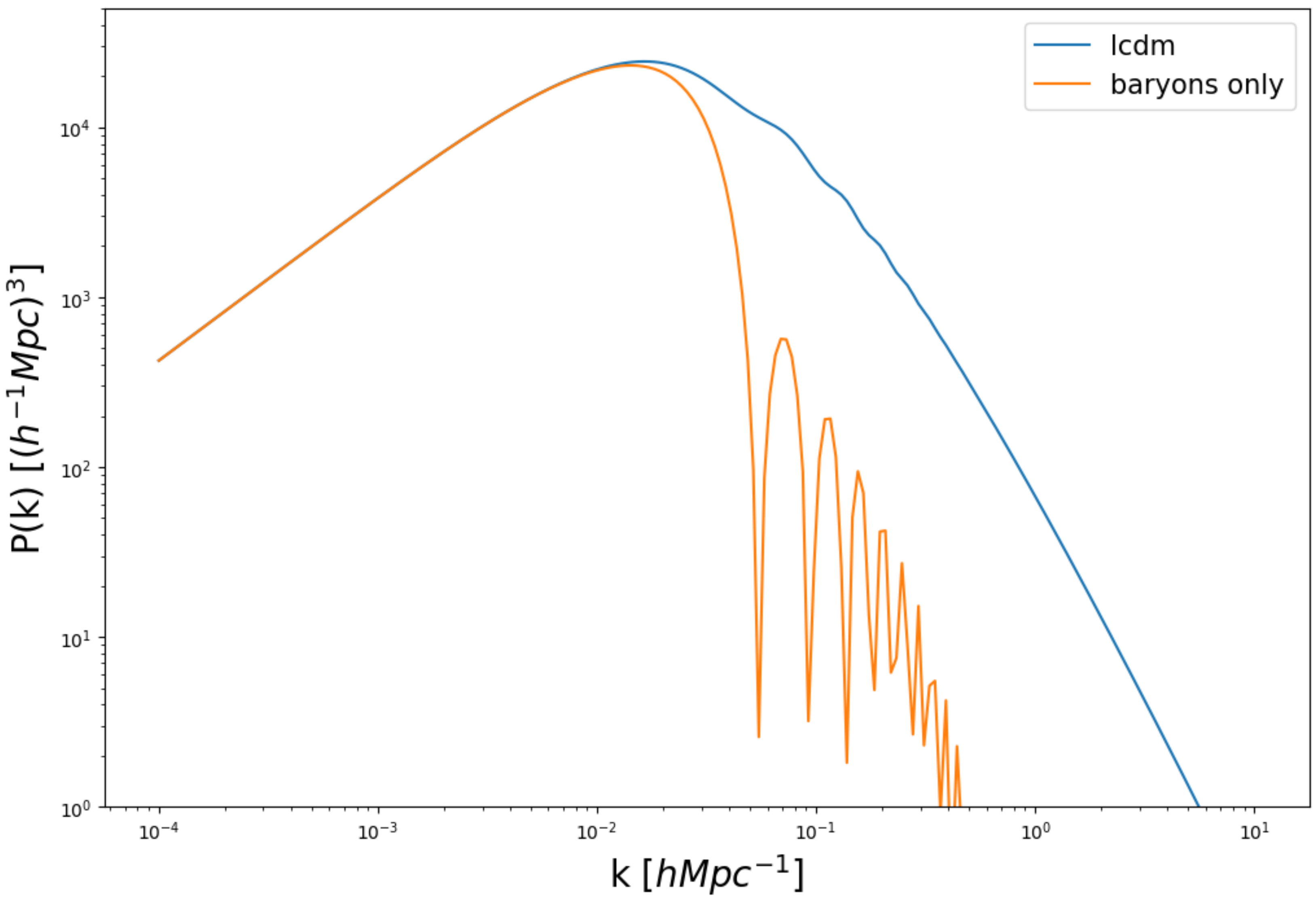


post-decoupling (z = 1100-100)

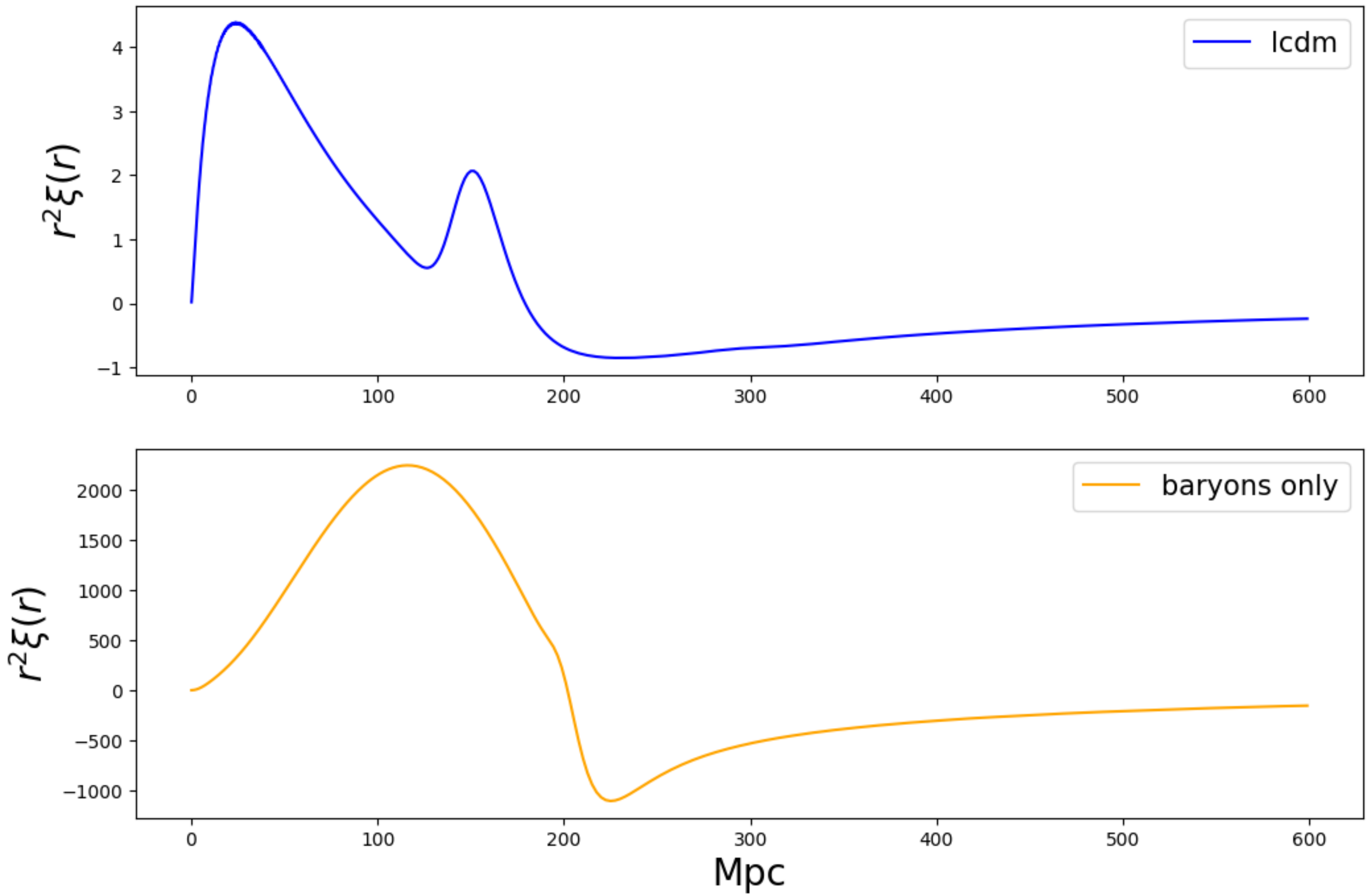
The simulations illustrate key facts about dark matter:

1) Dark matter do not evolve like baryonic matter, since it doesn't interact with photons.
2) Baryonic matter, after decoupling, falls into the DM gravitational potential wells.

If the Universe did not have any dark matter, the statistical distribution of structures today would look really different

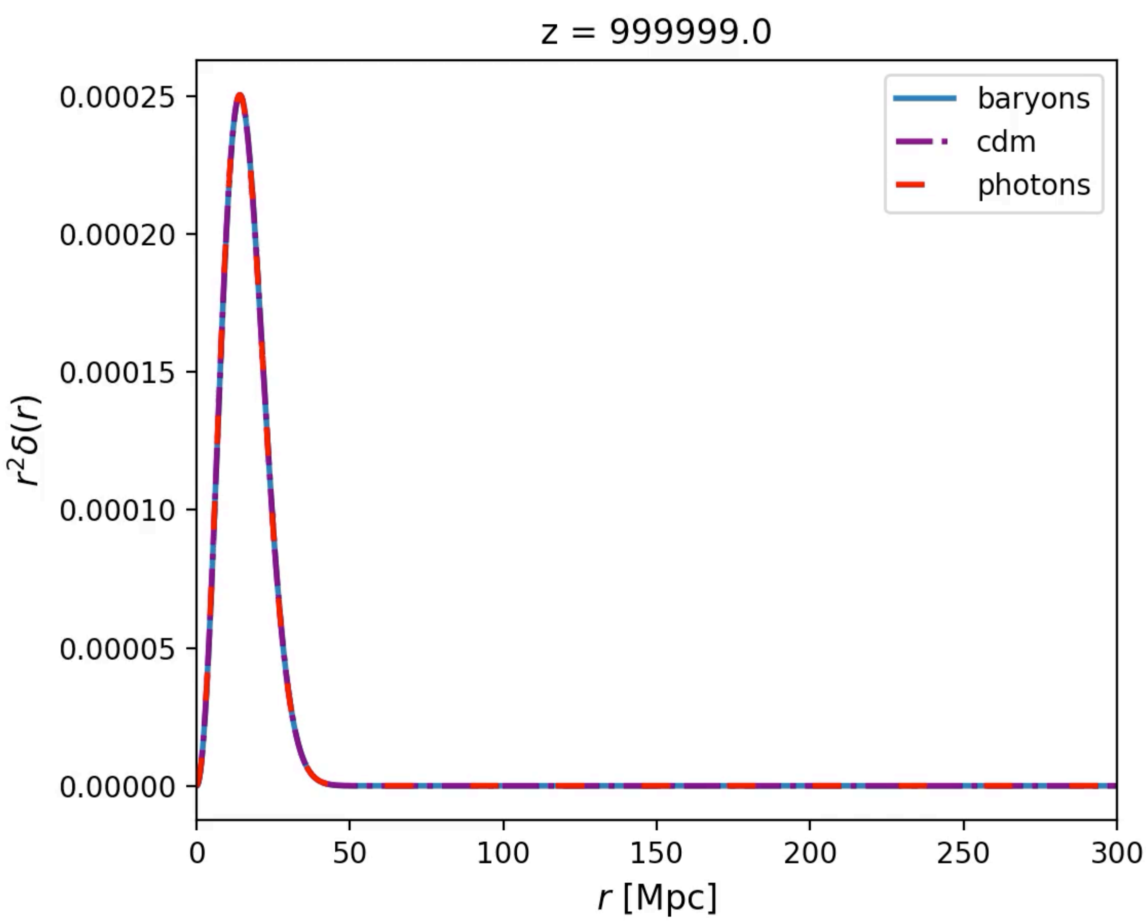


The real space equivalent to the matter power spectrum is the correlation function

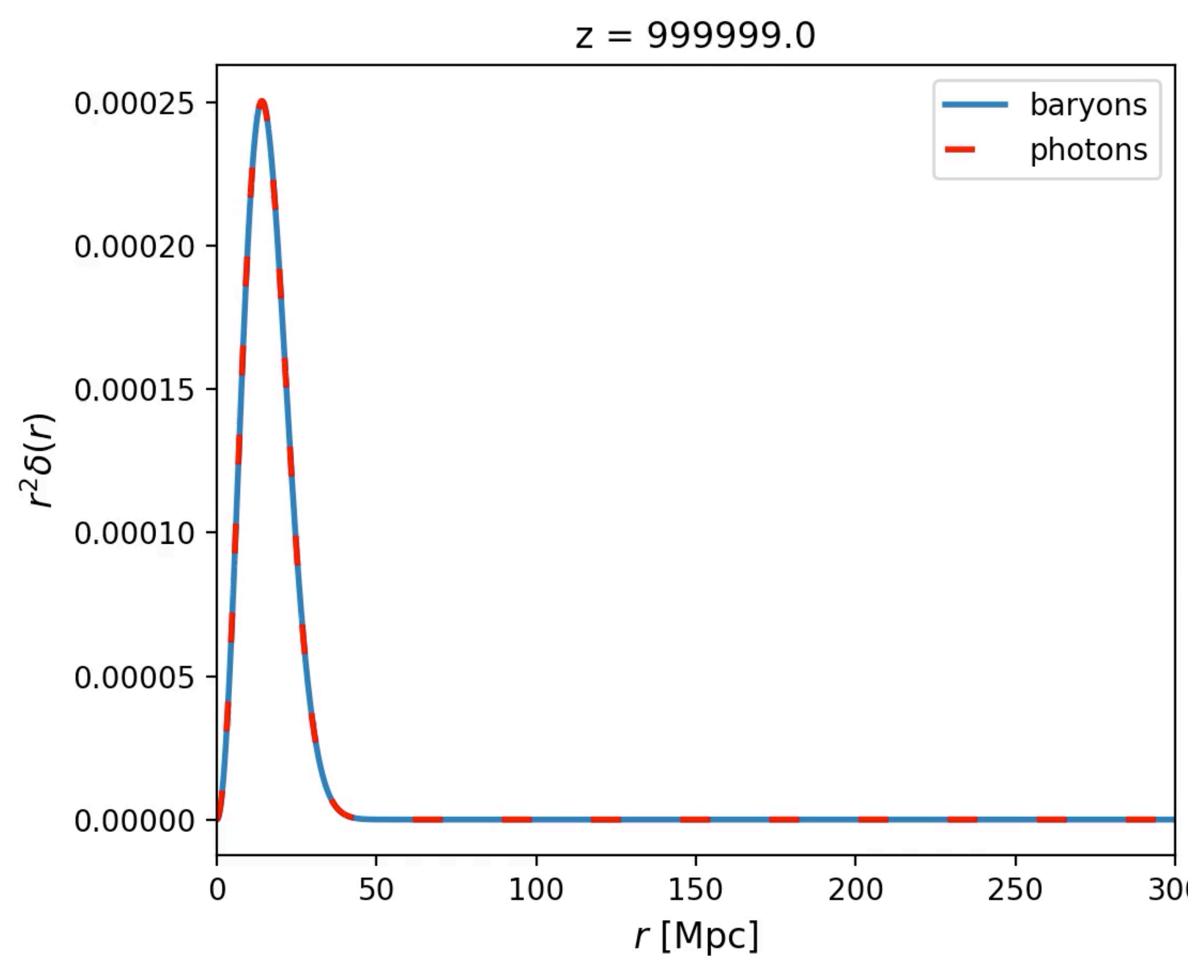


The real space correlation function physics is easy to understand if we go back to our real space animation focusing on a single perturbation

LCDM

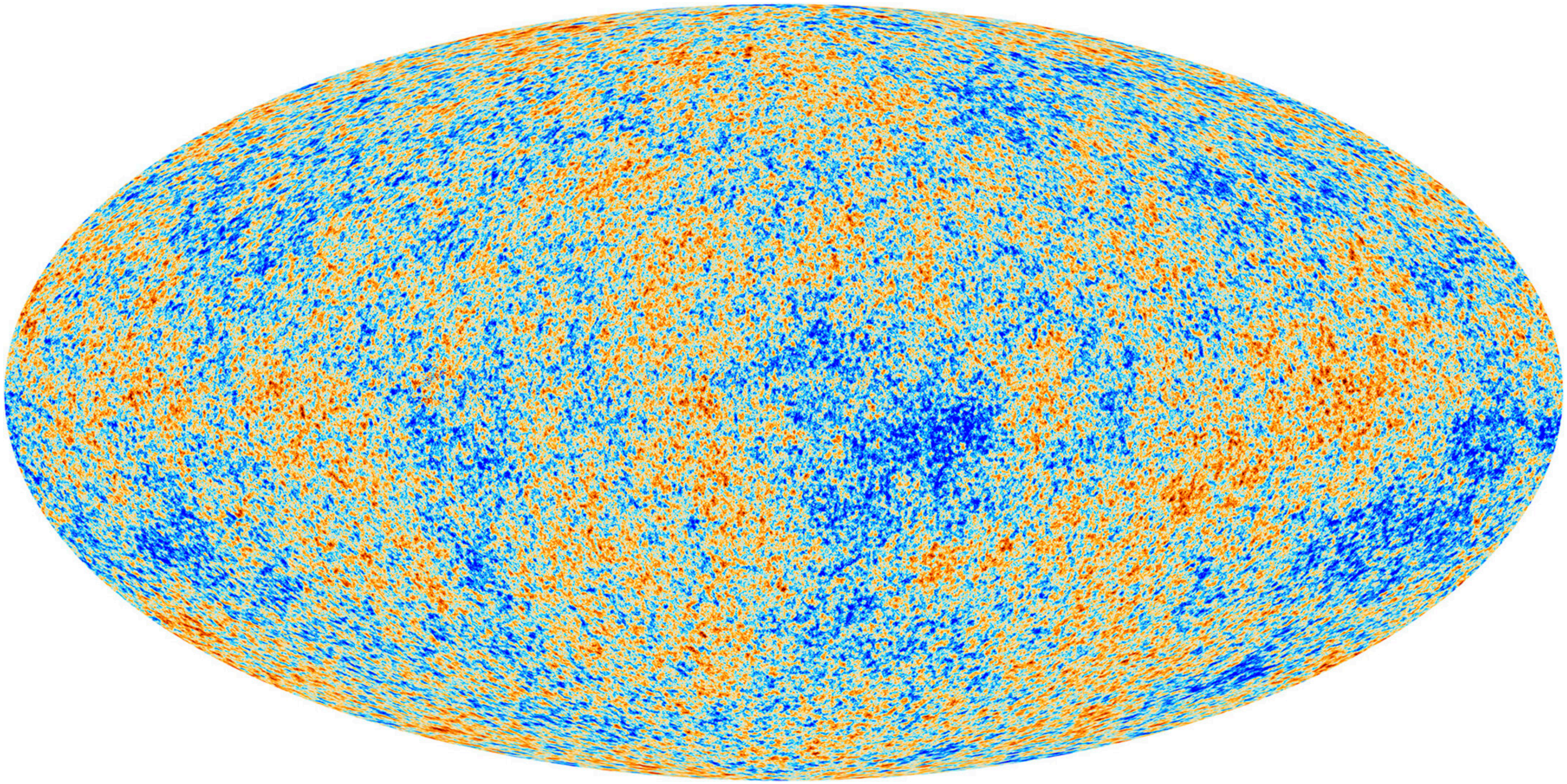


Baryons only



The CMB power spectrum

The CMB is observed on a sphere



Instead of the spatial power spectrum P(k) we describe its statistical properties using the angular power spectrum C_ℓ

The temperature anisotropies field at any point in space and time can be expanded In spherical harmonics

$$\Theta(\boldsymbol{x}, \hat{p}, \eta) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(\boldsymbol{x}, \eta) Y_{\ell m}(\hat{p})$$

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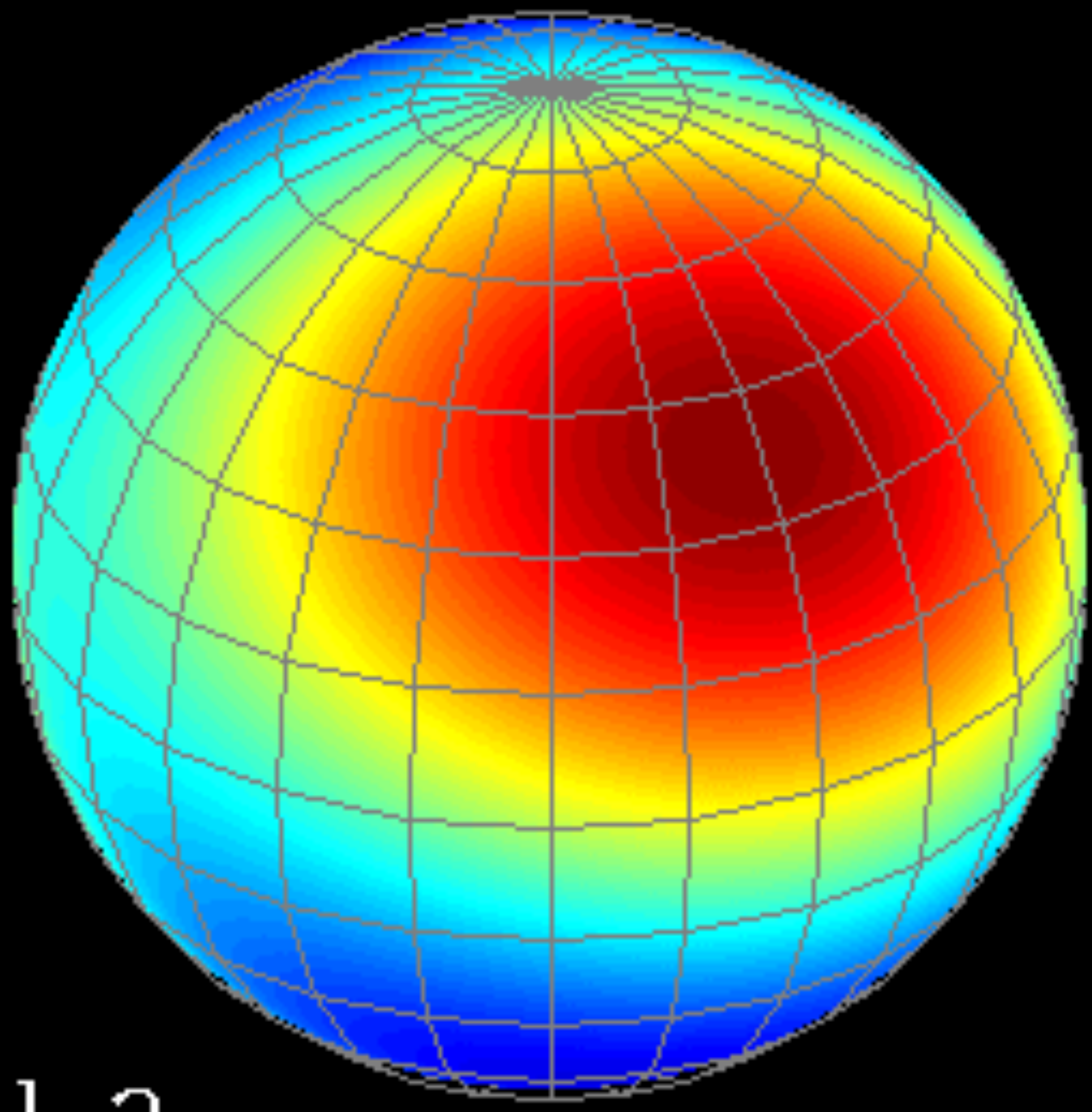
$$\Theta(\boldsymbol{x}, \hat{p}, \eta) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(\boldsymbol{x}, \eta) Y_{\ell m}(\hat{p})$$

This relation can be inverted, giving an expression for the alms as a function of
The temperature anisotropies field

$$a_{\ell m}(\boldsymbol{x}, \eta) = \int d\hat{p}\; Y^*_{\ell m}(\hat{p}) \Theta(\boldsymbol{x}, \hat{p}, \eta)$$

The angular power spectrum that will describe the statistical properties of the
field is defined as the variance of the $a_{\ell m}$

$$C_\ell(\boldsymbol{x}, \eta) = \langle a_{\ell m}(\boldsymbol{x}, \eta) a^*_{\ell m}(\boldsymbol{x}, \eta) \rangle$$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The formula for the CMB power spectrum look like the one of the matter power spectrum, we can write the CMB power spectrum as the product of two quantities

$$C_\ell = \frac{2}{\pi}\int_0^\infty dk k^2 P_{\mathcal{R}}(k)|\mathcal{T}_\ell(k)|^2$$

The primordial power spectrum

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3}A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$

A transfer function that takes into account the evolution of perturbation in the universe

$$C_\ell = \frac{2}{\pi}\int_0^\infty dk k^2 P_{\mathcal{R}}(k)|\mathcal{T}_\ell(k)|^2$$

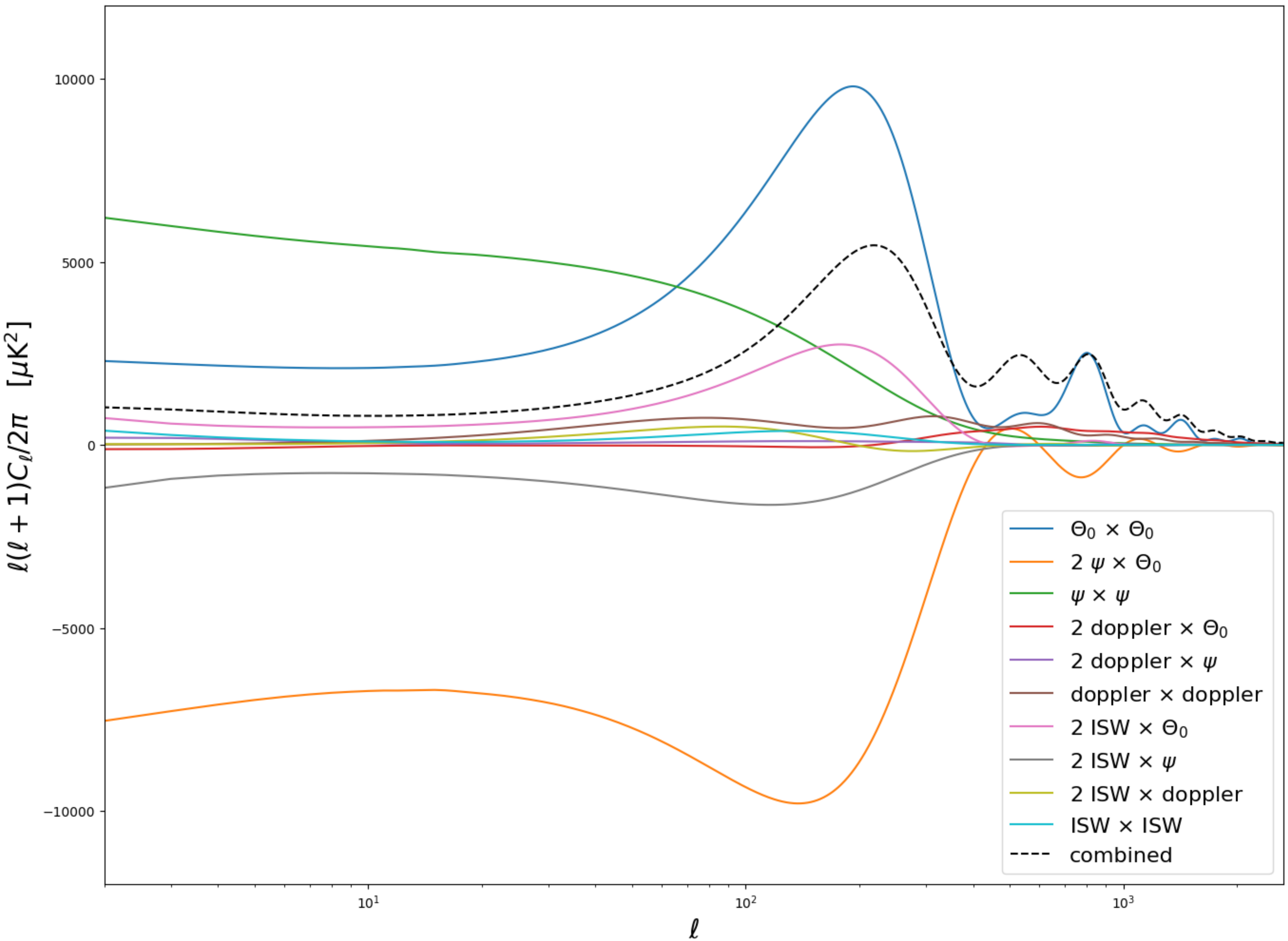
The form of $\mathcal{T}_\ell(k)$ follows from the fundamental equation for CMB anisotropy

$$\boxed{\Theta|_{\eta_R} = \int_0^{\eta_R} d\eta g(\eta)\left[(\Theta_0+\psi)+\boldsymbol{e}\boldsymbol{u}_b\right] + \int_0^{\eta_R} d\eta \exp(-\tau)(\dot{\psi}-\dot{\phi})}$$

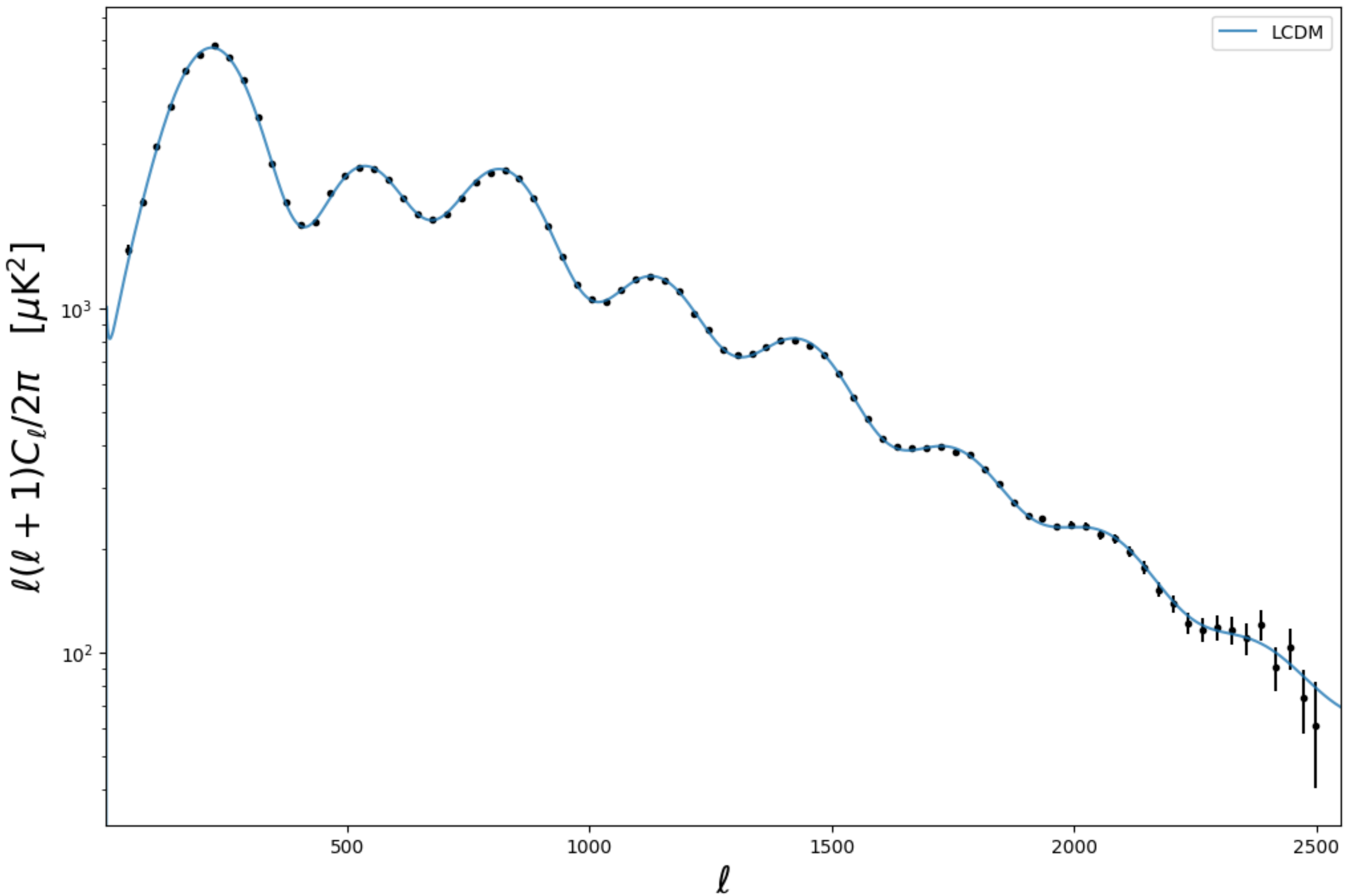
$$\downarrow$$

$$\begin{aligned}
\mathcal{T}_\ell(k) \;\; &\propto \;\; \int_0^{\eta_R} d\eta g(\eta)\left[\Theta_0(k,\eta)+\psi(k,\eta)\right] j_\ell[k(\eta-\eta_R)] \\
&- \;\; \frac{i}{k}\int_0^{\eta_R} d\eta g(\eta) u_b(k,\eta)\frac{d}{d\eta} j_\ell[k(\eta-\eta_R)] \\
&+ \;\; \int_0^{\eta_R} d\eta e^{-\tau(\eta)}\left[\psi'(k,\eta)-\phi'(k,\eta)\right] j_\ell[k(\eta-\eta_R)]
\end{aligned}$$

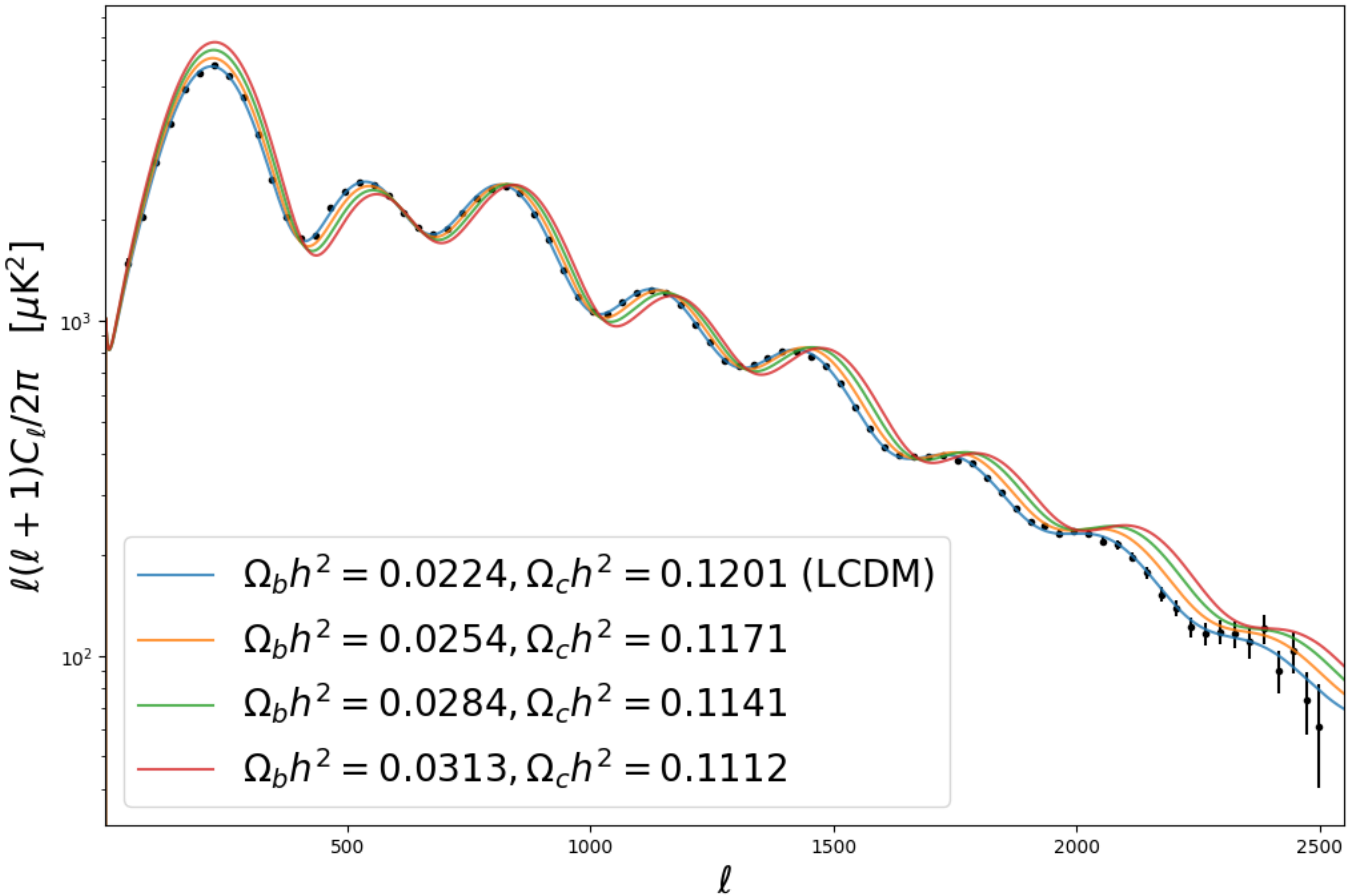
Each of these effects contribute to the total CMB power spectrum



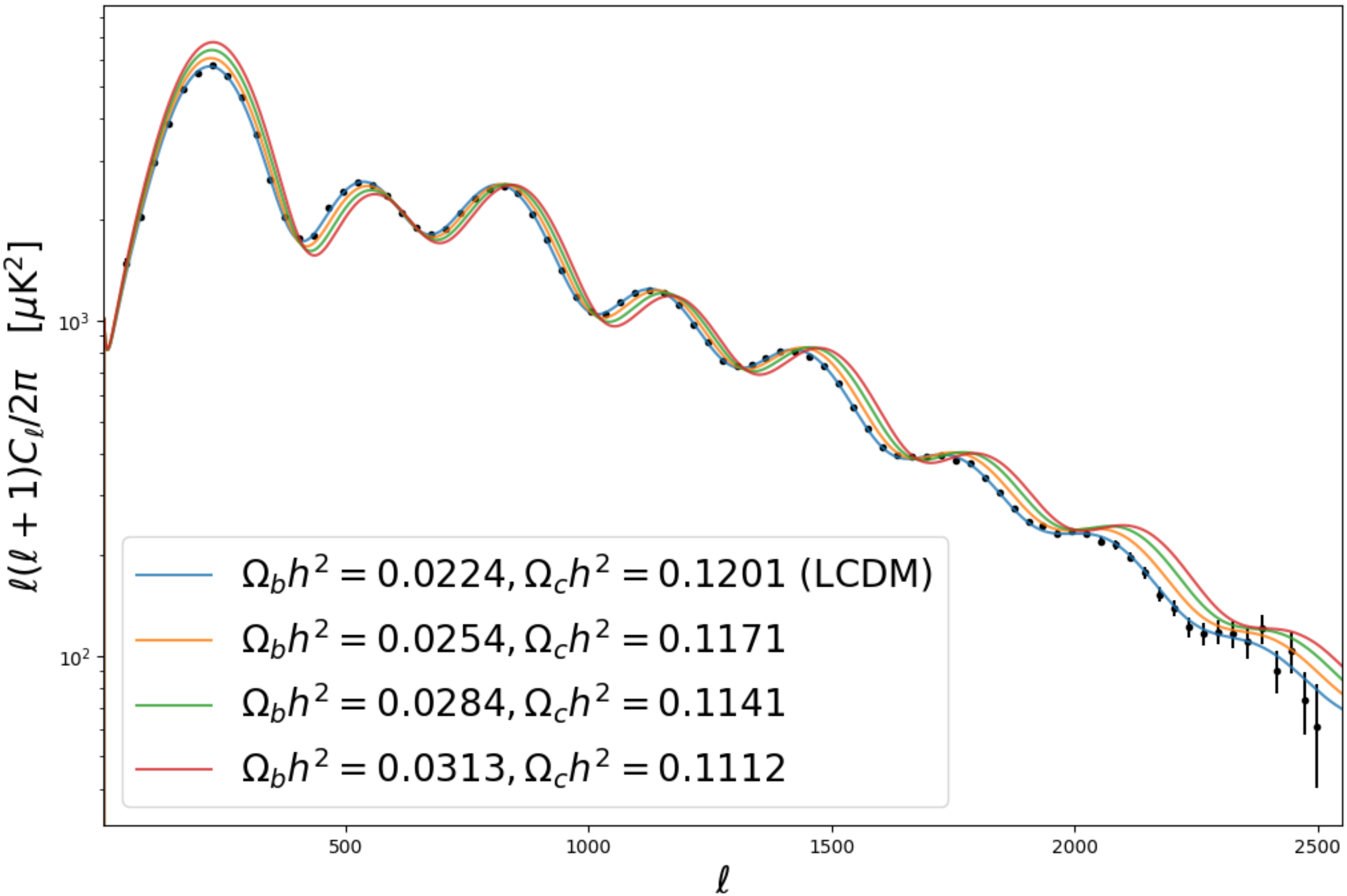
The CMB power spectrum has been extremely well measured by the Planck satellite



We can look at the effect of increasing the baryon density and decreasing the Dark matter density

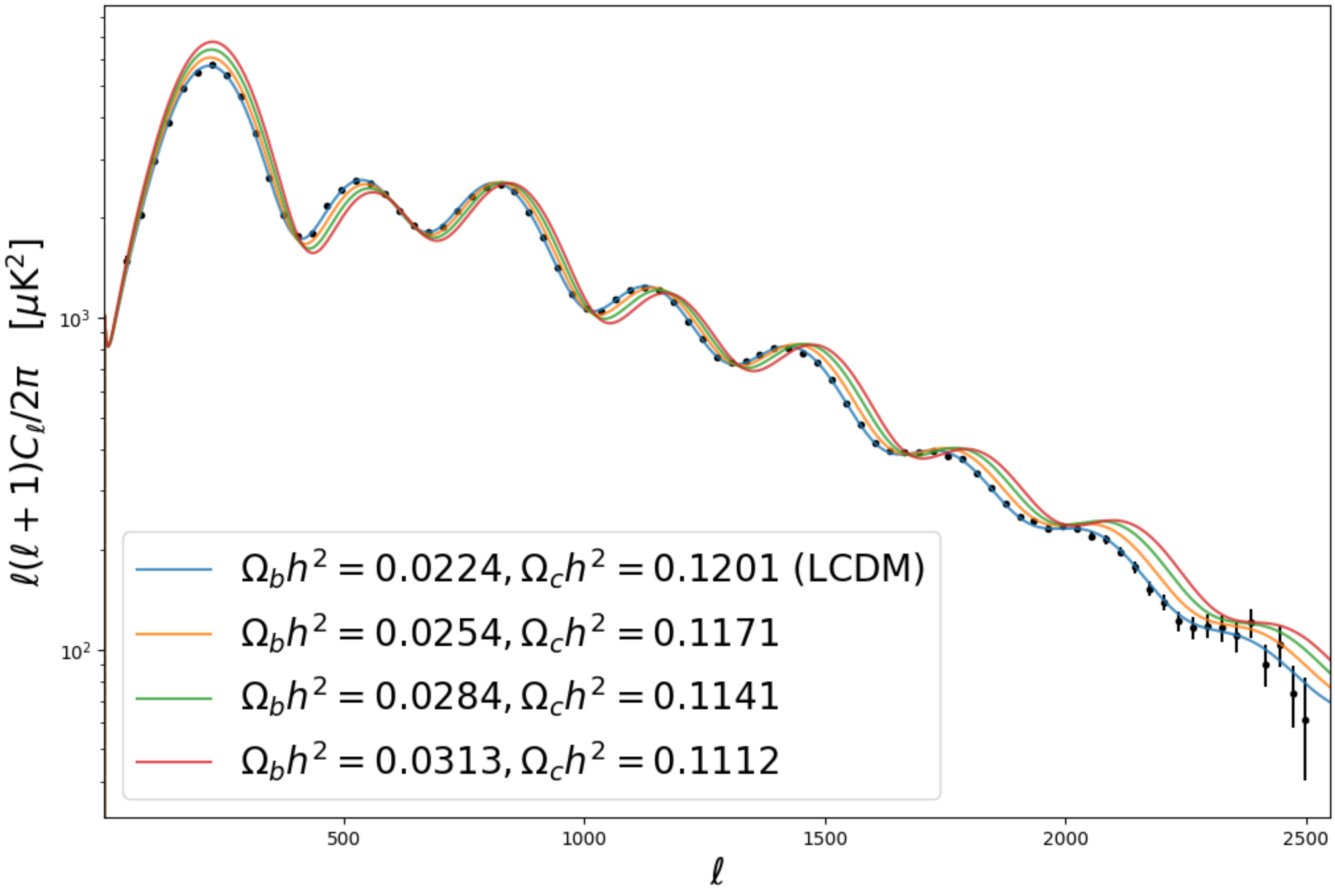


Two main effects: 1) When we increase the baryon density the peak are shifted to the right



Two main effects: 1) When we increase the baryon density the peak are shifted to the right

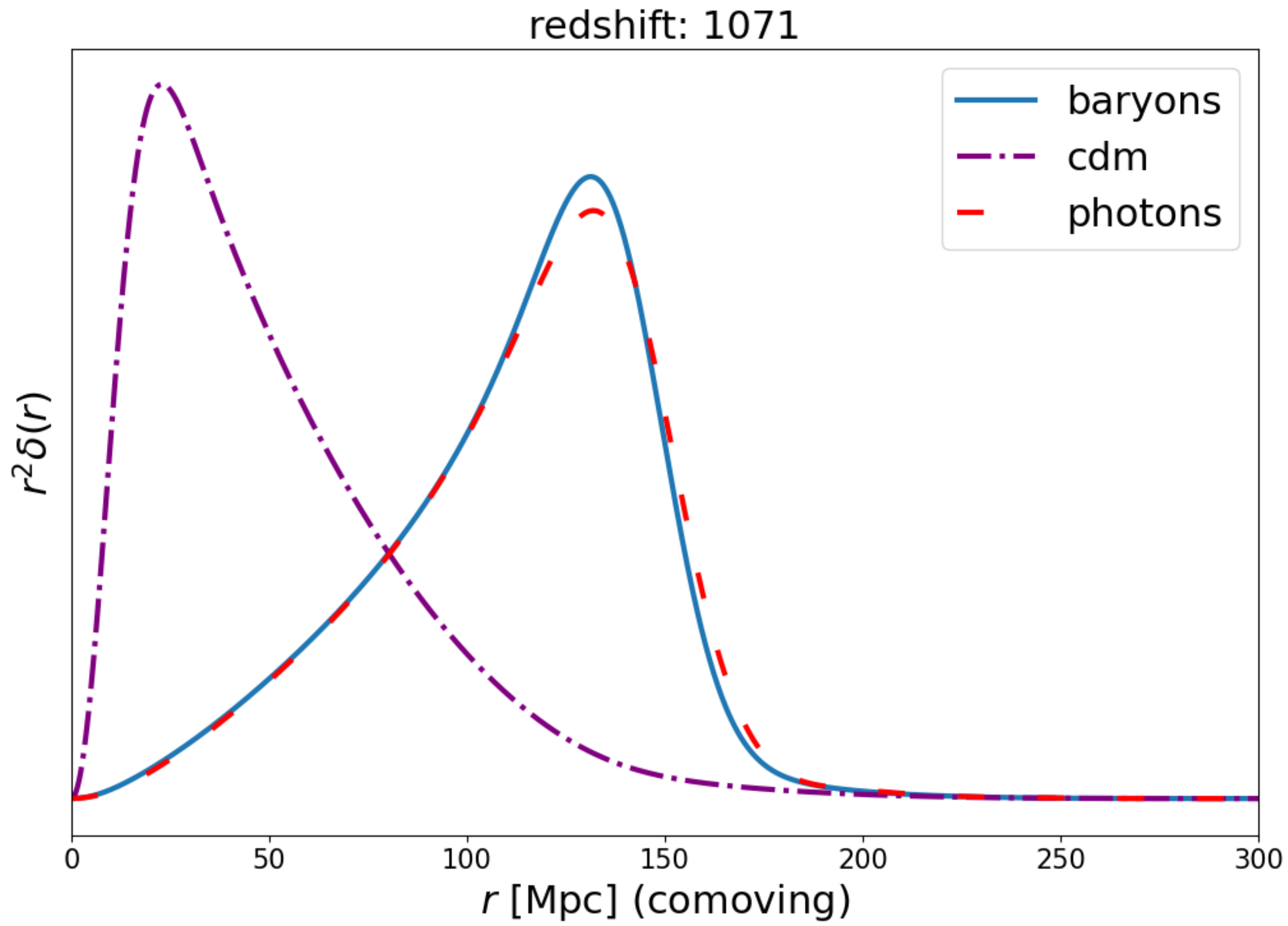
2) When we increase the baryons density there is more power On small scales



1) When we increase the baryon density the peak are shifted to the right

The location of the peak in the power spectrum is related to what is called the Sound horizon: it is the distance the baryon-photon wave can propagate upon, from its generation to decoupling

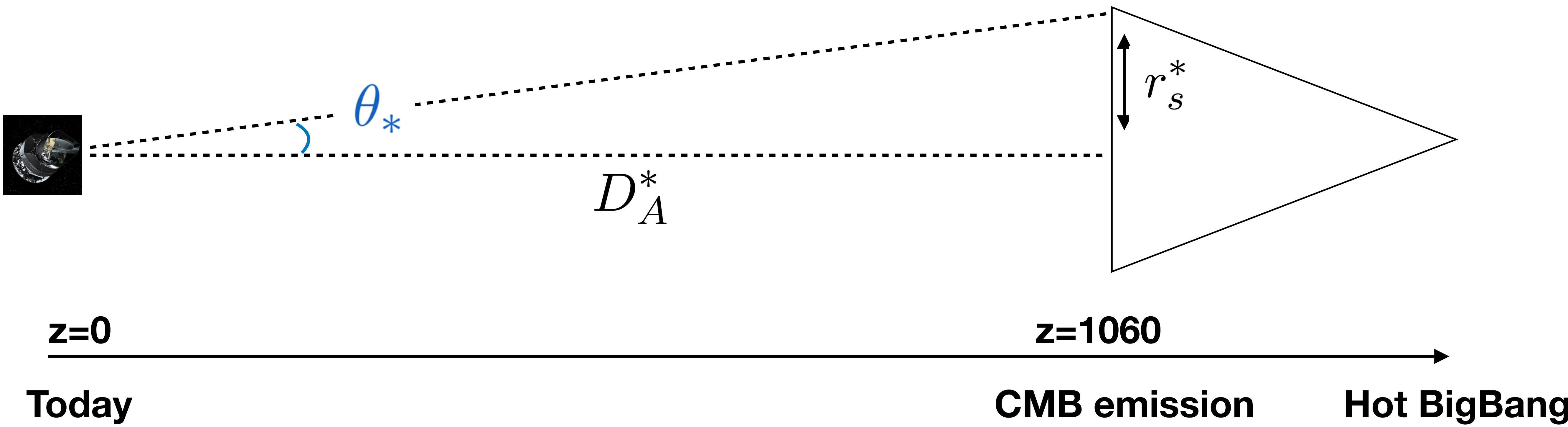
In LCDM, this distance is of order 150 Mpc (comoving)



1) When we increase the baryon density the peak are shifted to the right

This distance can be transformed to an angle on the sphere

$$\theta^* = \frac{r_s^*}{D_A^*}$$



Where D_A^* is the distance to the last scattering surface

1) When we increase the baryon density the peak are shifted to the right

A feature of angular size θ_* will produce a set of harmonics at location

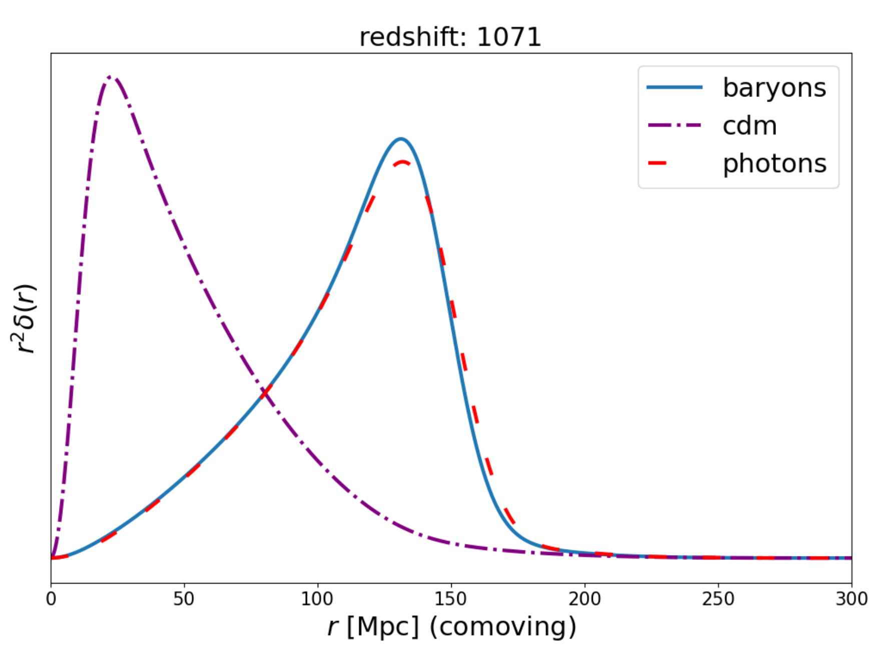
$$\ell_n^* \sim n\frac{\pi}{\theta*}$$

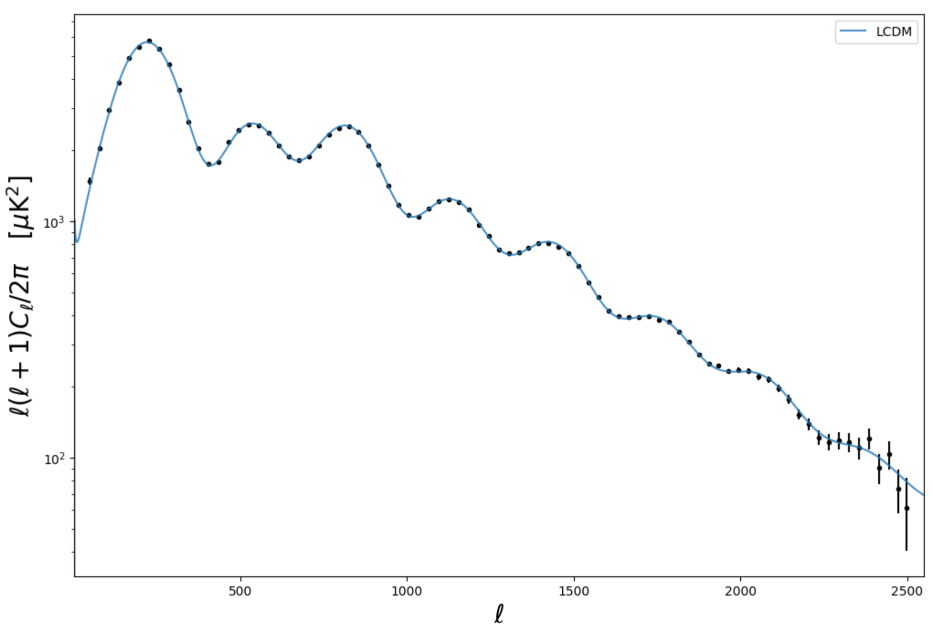
The whole chain of reasoning is therefore:

Photons-baryons wave propagate a distance r_s^*

Which correspond to an angle on the last scattering surface of $\theta^* = \dfrac{r_s^*}{D_A^*}$

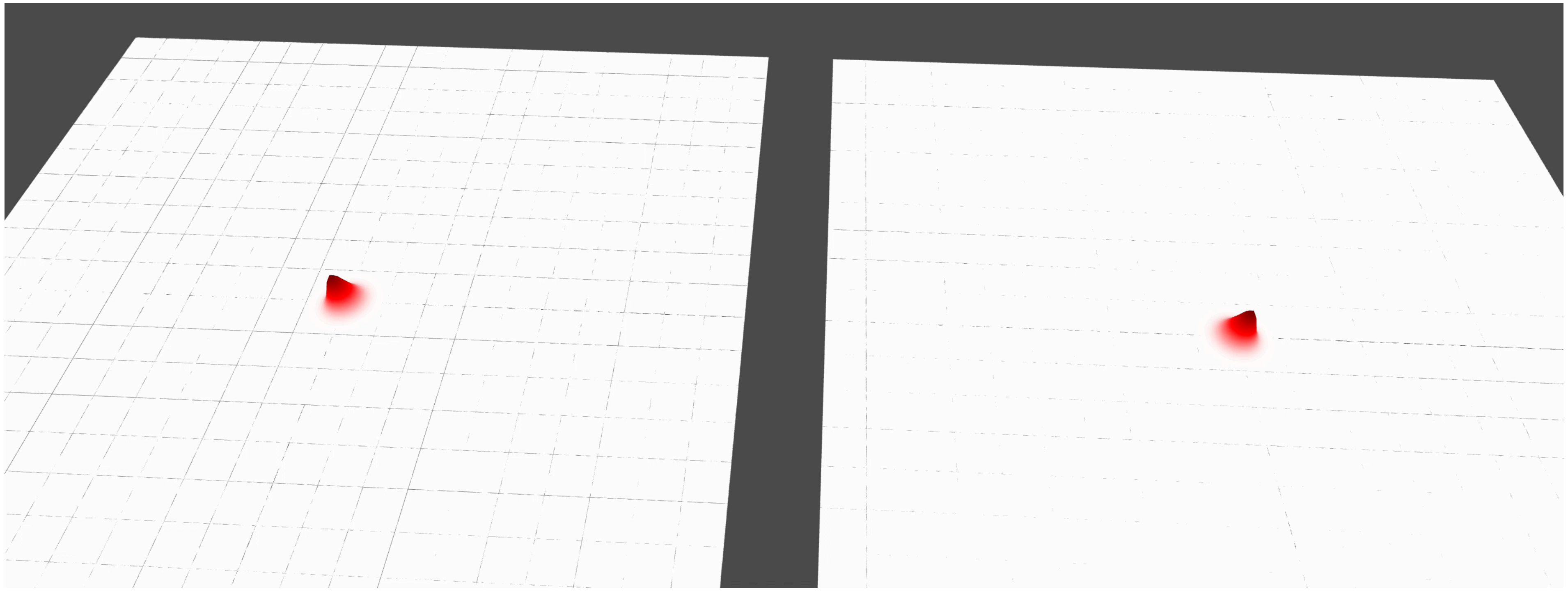
Which correspond to a set of peak at multipole location $\ell_n^* \sim n\dfrac{\pi}{\theta*}$





1) When we increase the baryon density the peak are shifted to the right

Ok, so what happens when you increase the baryon density (and decrease the dark matter density)

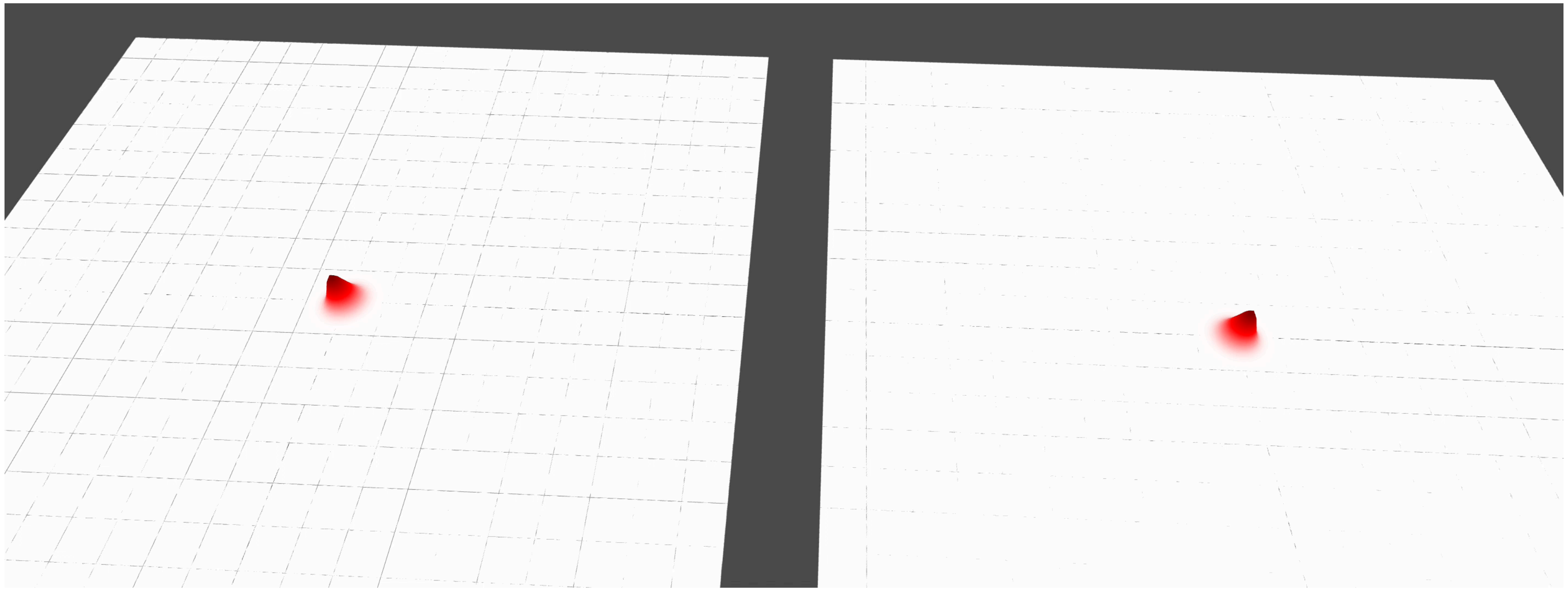


LCDM

High baryons- low dark matter

1) When we increase the baryon density the peak are shifted to the right

Ok, so what happens when you increase the baryon density (and decrease the dark matter density)



LCDM

High baryons- low dark matter

The baryons-photons wave does not propagate as far

1) When we increase the baryon density the peak are shifted to the right

In order to understand this we can write down a formula for the sounds horizon

$$
\begin{aligned}
r_s^* &= \int_0^{t^*} \frac{dt}{a(t)} c_s(t) \\
&= \int_{z^*}^{\infty} \frac{dz}{H(z)} c_s(z) \\
&= \int_{z^*}^{\infty} \frac{dz}{H(z)} \sqrt{\frac{1}{3\left[1+\frac{3\rho_b}{4\rho_\gamma}\right]}}
\end{aligned}
$$

The baryons gives inertia to the plasma, more baryons slow down the waves

1) When we increase the baryon density the peak are shifted to the right

In order to understand this we can write down a formula for the sounds horizon

$$\begin{aligned} r_s^* &= \int_0^{t^*} \frac{dt}{a(t)} c_s(t) \\ &= \int_{z^*}^{\infty} \frac{dz}{H(z)} c_s(z) \\ &= \int_{z^*}^{\infty} \frac{dz}{H(z)} \sqrt{\frac{1}{3\left[1+\frac{3\rho_b}{4\rho_\gamma}\right]}} \end{aligned}$$

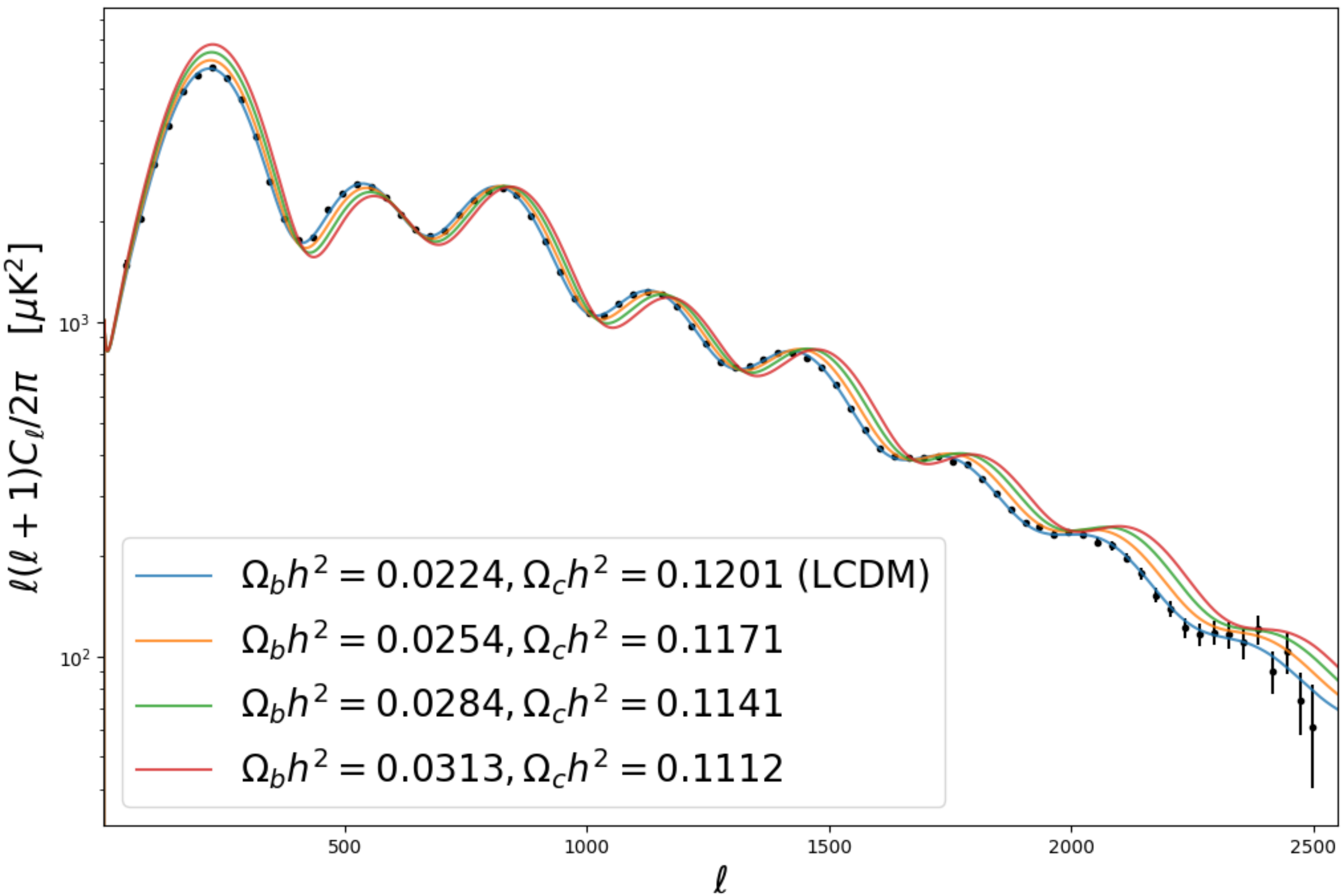
The baryons gives inertia to the plasma, more baryons slow down the waves

$$\theta^* = \frac{r_s^*}{D_A^*} \qquad \ell_n^* \sim n\frac{\pi}{\theta *}$$

$$\rho_b \nearrow, \quad c_s \searrow, \quad r_s^* \searrow, \quad \theta^* \searrow, \quad \ell_n^* \nearrow$$

1) When we increase the baryon density the peak are shifted to the right

$$\rho_b \nearrow, \quad c_s \searrow, \quad r_s^* \searrow, \quad \theta^* \searrow, \quad \ell_n^* \nearrow$$



2) When we increase the baryon density there is more power on small scales

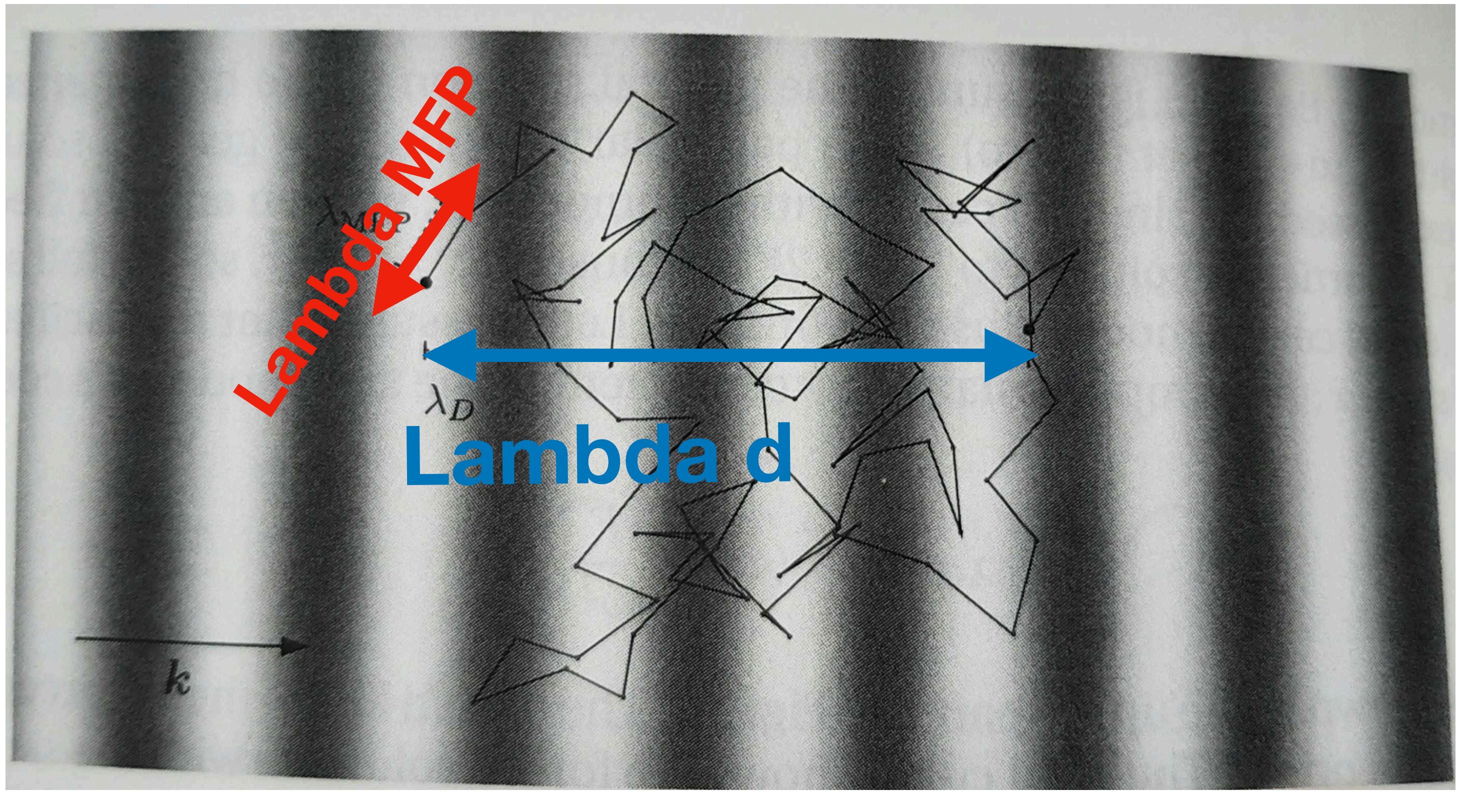
On small scales an important effect comes from photons diffusion, in the early universe baryons and photons behave like a fluid, the mean free path of a photon can be approximated as being basically zero, when the CMB is emitted the mean free path is infinite (photon do not interact with baryons anymore).

In between, there is transition where the mean free path is neither zero, neither infinite, they diffuse on short distance.

The effect of diffusion is to smooth out the universe, perturbation smaller than the diffusion scale will be reduced since diffusion can mix hot and cold spot together

2) When we increase the baryon density there is more power on small scales

Consider the path of a single photon as it scatters off a sea of electrons.



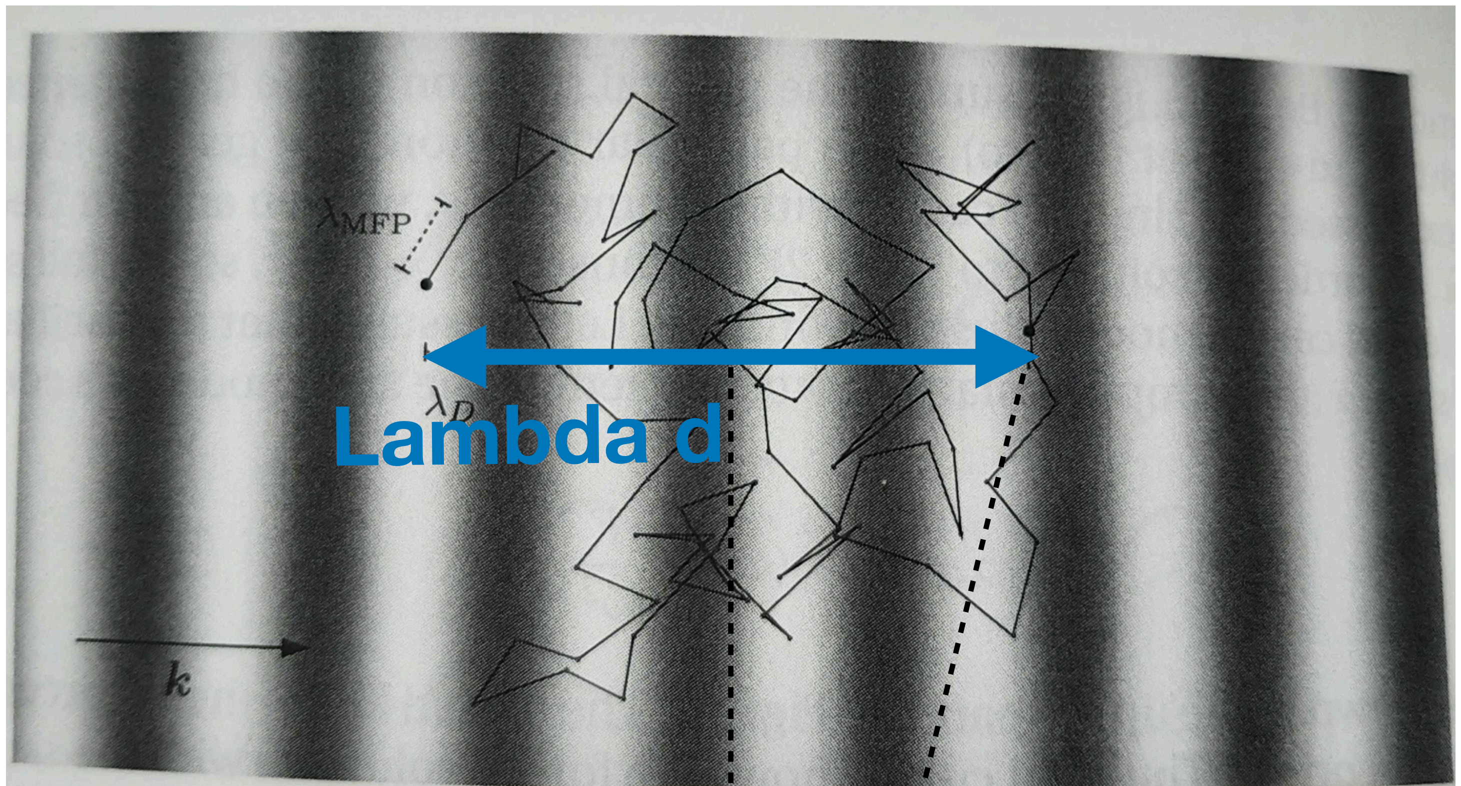
The comoving mean free path of a photon is given by

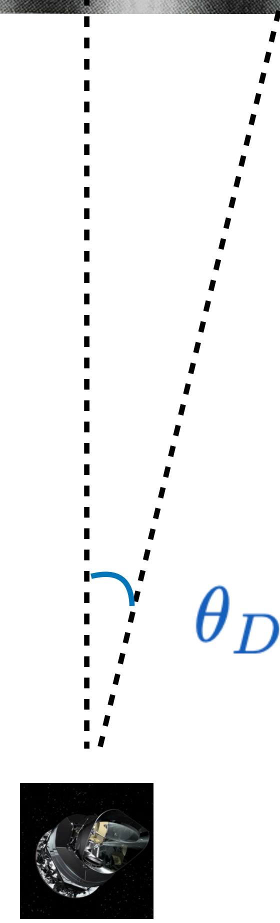
$$\lambda_{\text{MFP,comoving}} = (a n_e \sigma_T)^{-1}$$

Using Brownian motion theory, we can compute how far how photon propagates After their numerous scatters with electrons λ_D

$$\lambda_D \sim \frac{1}{\sqrt{n_e \sigma_T H}} \frac{1}{a}$$

2) When we increase the baryon density there is more power on small scales





$$\theta_D \propto \frac{\lambda_D}{D_A^*}$$

$$\ell_D \propto \frac{\pi}{\theta_D}$$

Scales with $\ell > \ell_D$ will therefore be suppressed

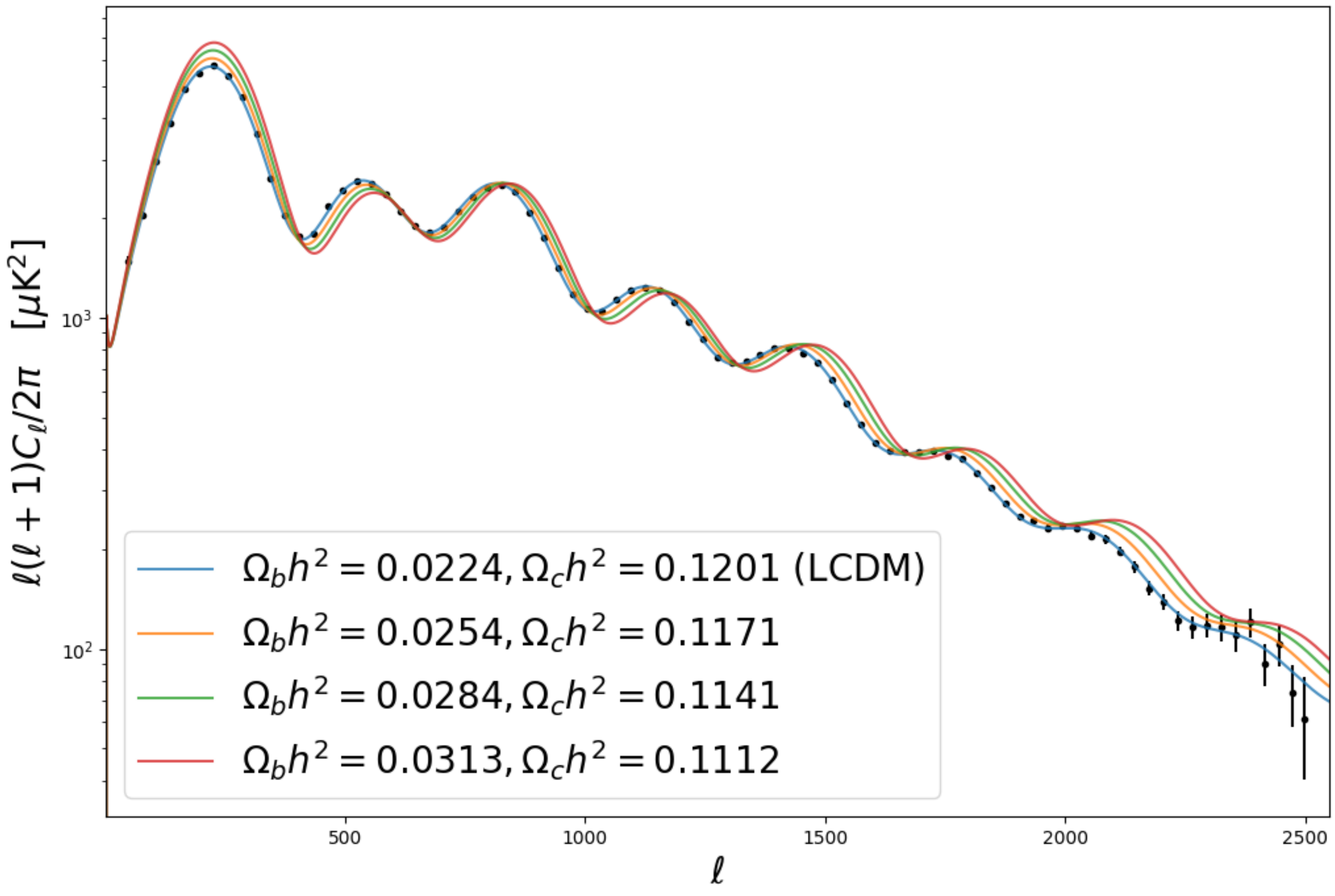
2) When we increase the baryon density there is more power on small scales

$$\lambda_D \sim \lambda_{\mathrm{MFP,comoving}} \sqrt{n_e \sigma_T H^{-1}} = \frac{1}{\sqrt{n_e \sigma_T H}} \frac{1}{a}$$

$$\theta_D \propto \frac{\lambda_D}{D_A^*}$$

$$\rho_b \nearrow, \quad n_e \nearrow, \quad \lambda_D \searrow, \quad \theta_D \searrow, \quad \ell_D \nearrow$$

$$\ell_D \propto \frac{\pi}{\theta_D}$$



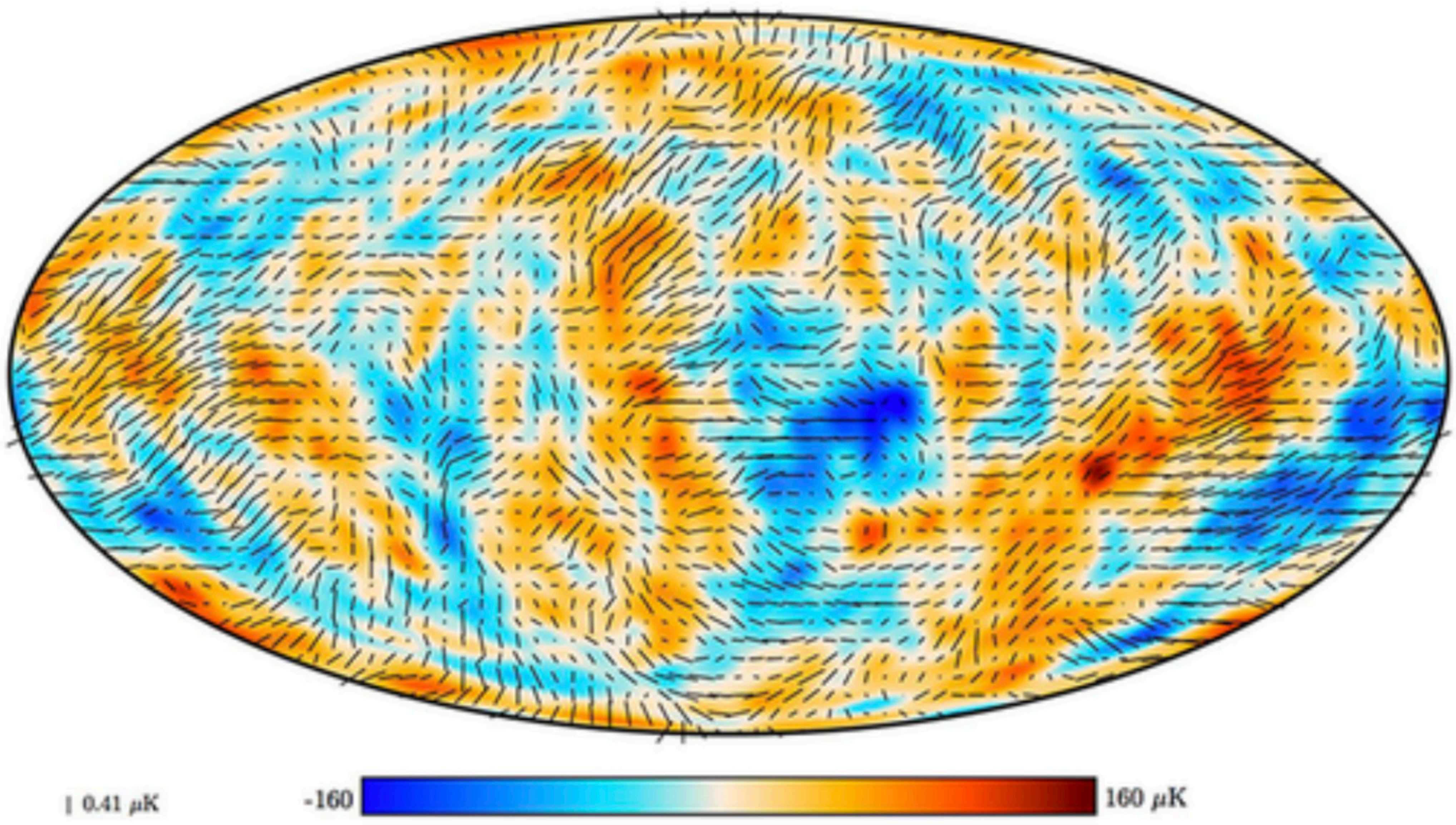
The main point here is that the CMB power spectrum behave differently if matter is composed of baryonic or dark matter.

The CMB temperature power spectrum is also only part of the information that we can obtain by mapping the micro-wave sky. Two other important observables are

1) The CMB polarisation

2) The gravitational lensing of the CMB

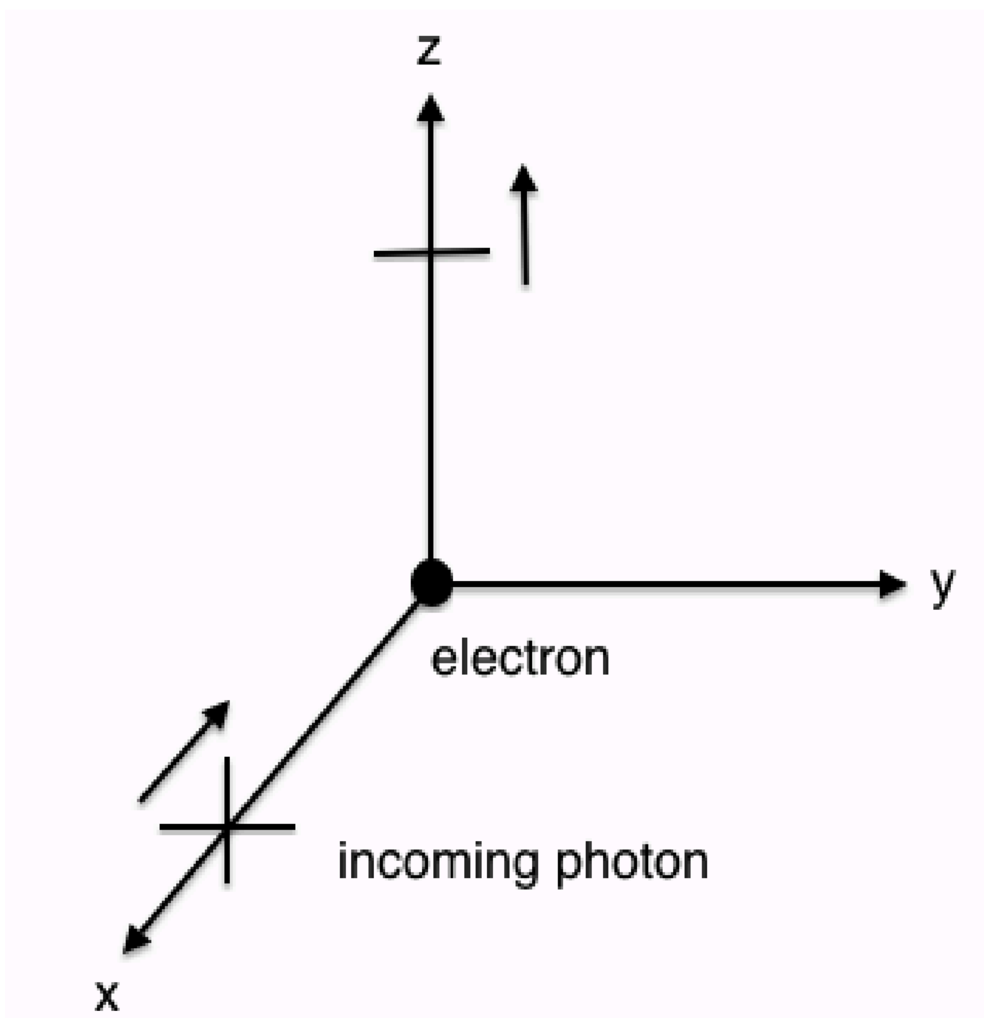
1) The CMB polarisation



The CMB is polarized, meaning that when we observe the electric fields associated with photons coming from a given direction on the sky, they tend to align in the same direction.

To understand this let's imagine a electron on the last scattering surface and look at how it interact with photons

First let's imagine scattering with a single photon

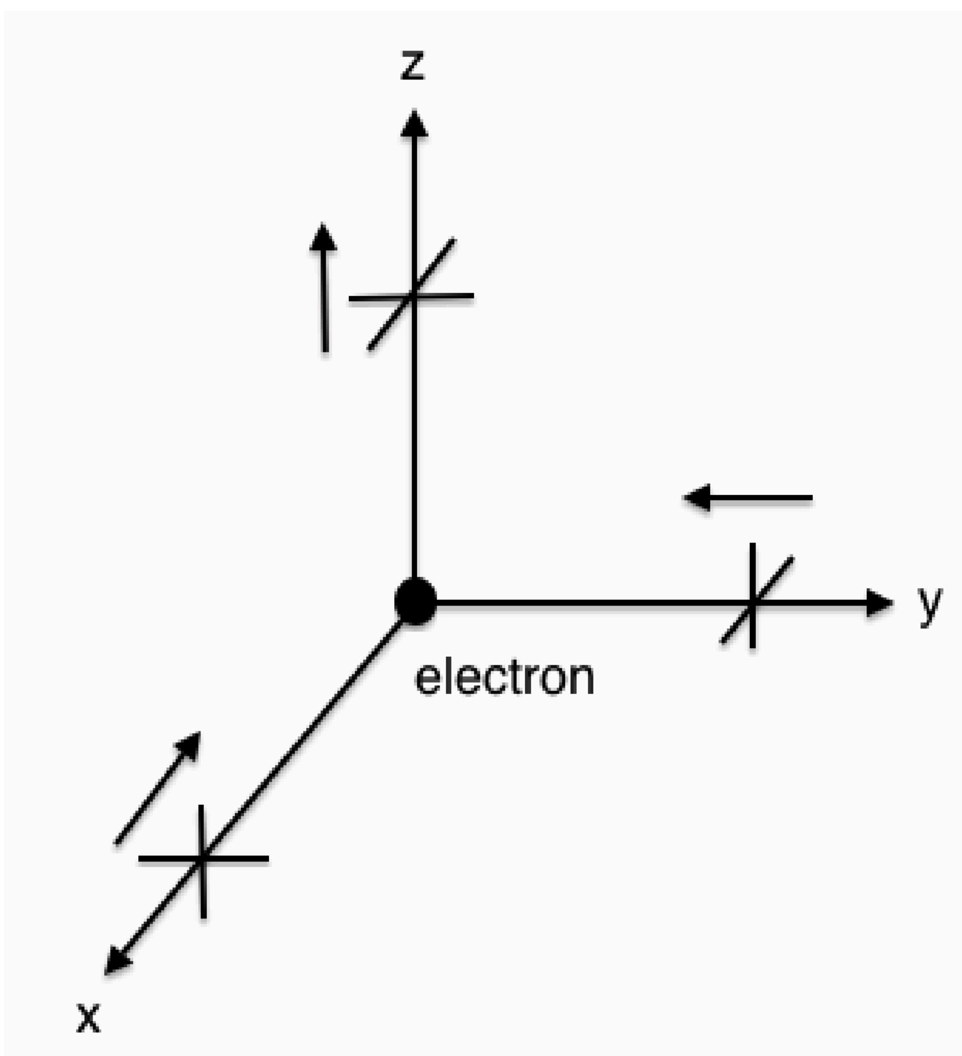


The represents the intensity

of polarisation in the z and y direction.

To understand this let's imagine a electron on the last scattering surface and look at how it interact with photons

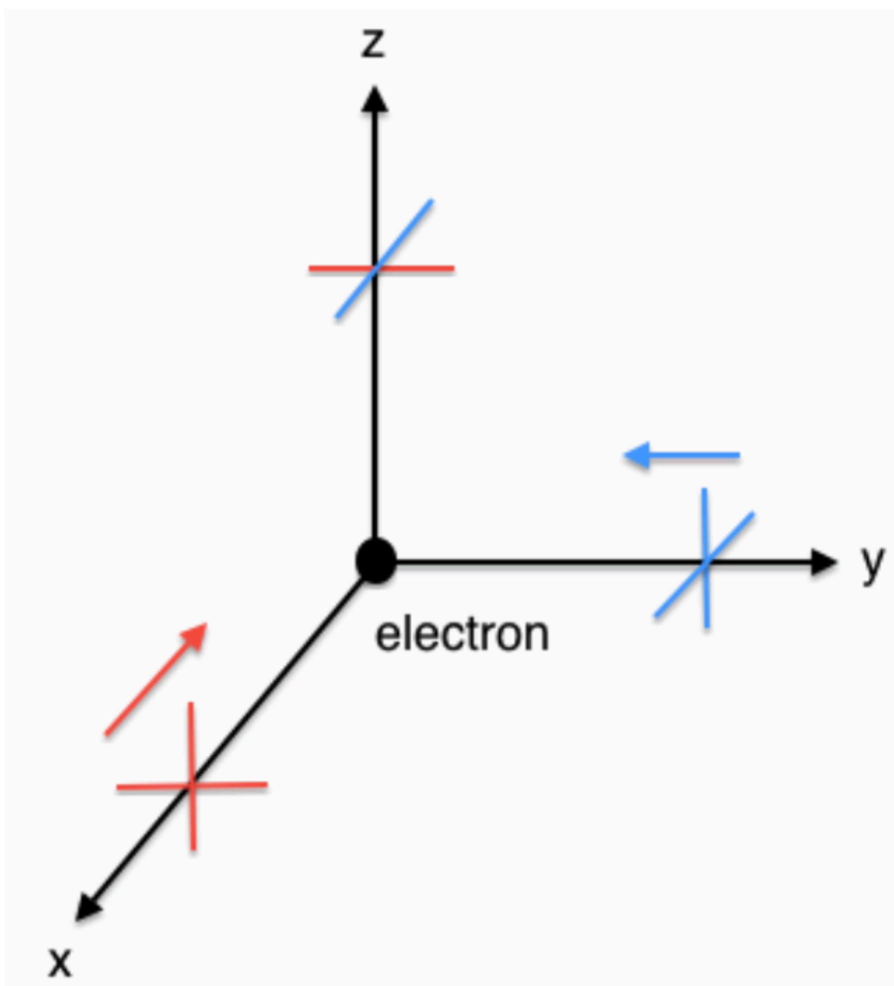
Now let's imagine the result of the scattering of an isotropic radiation field



No net polarisation is produced

To understand this let's imagine a electron on the last scattering surface and look at how it interact with photons

let's imagine the result of the scattering with a radiation field with a quadrupolar distribution

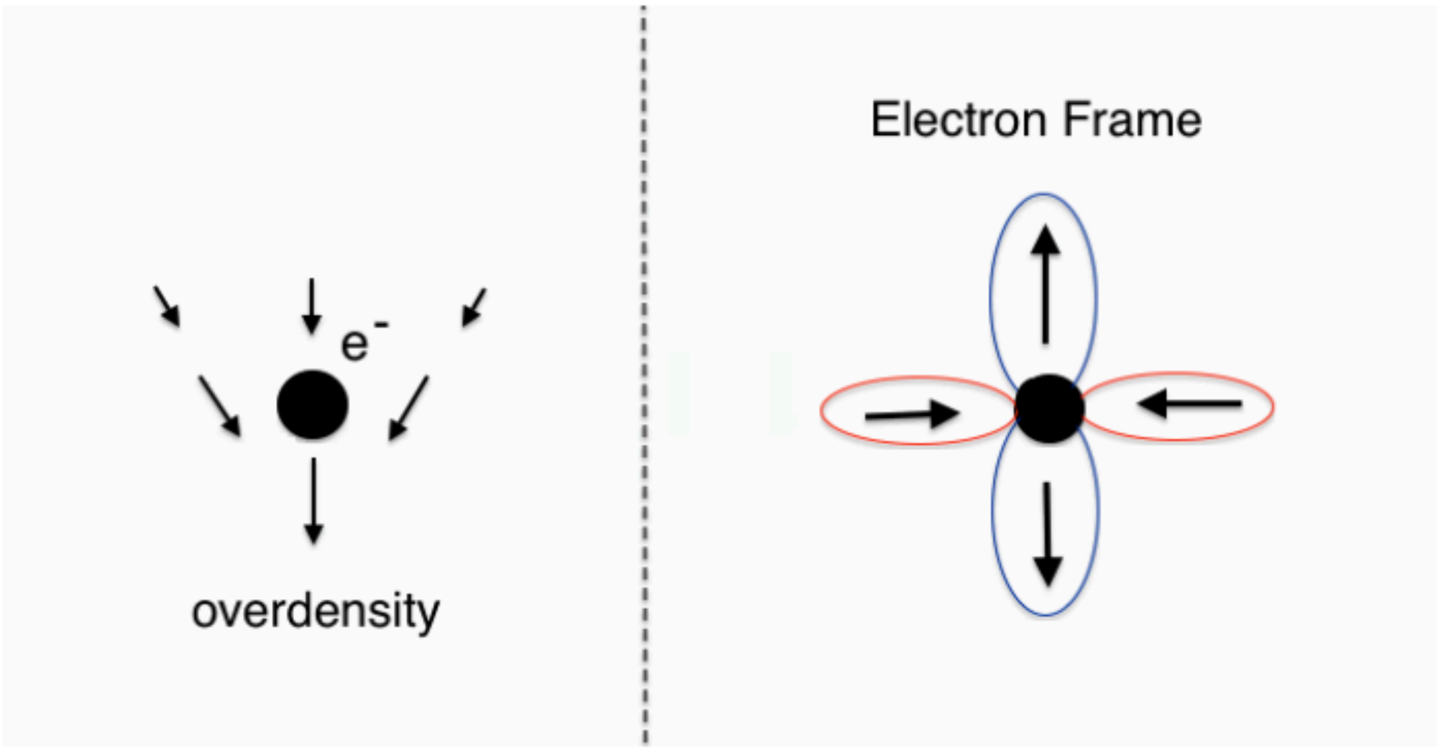


Polarisation is generated !

1) The CMB polarisation

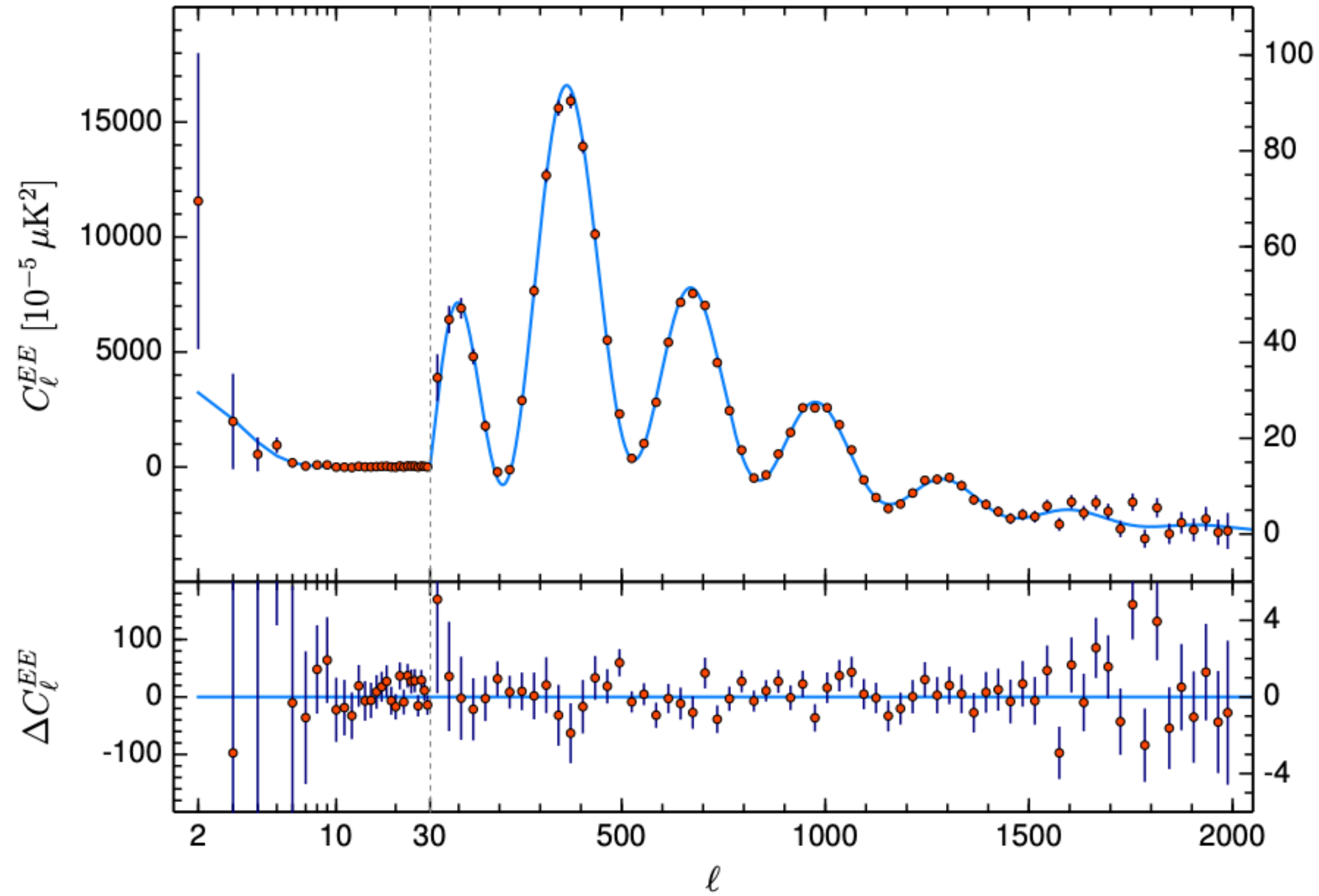
So the way to generate polarisation on the last scattering is though Compton scattering of a bunch of photons on electrons, and the polarisation amplitude will be fully determined by the local quadrupole of the incoming radiation.

a) One way to generate a quadrupole is simply from tidal forces, let's imagine Plasma falling into a potential well generated by an overdensity.

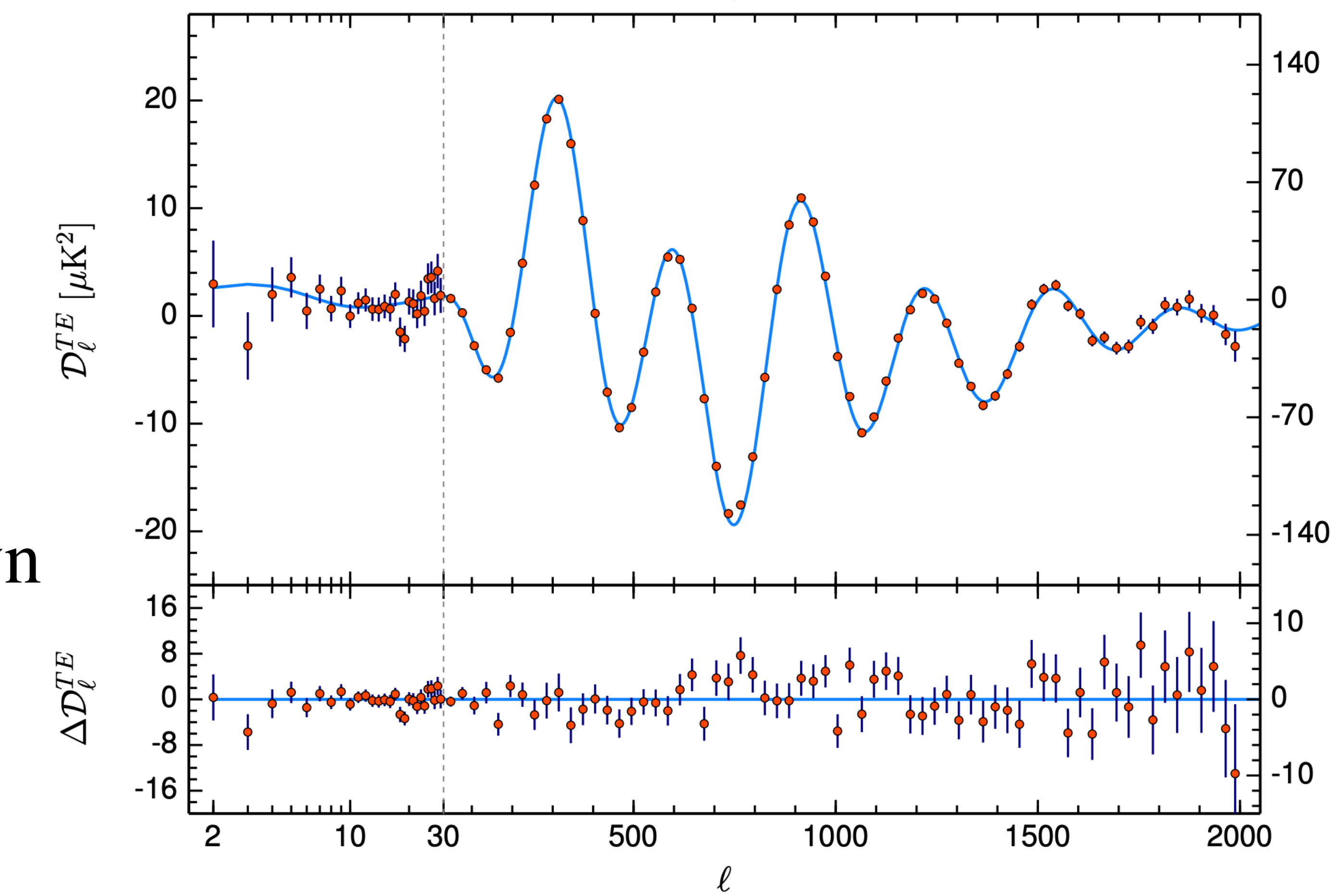


In the electron frame, the plasma velocity has a quadrupolar distribution,
Due to the standard doppler effect, the quadrupolar distribution of the plasma velocity becomes a quadrupolar distribution in the incoming radiation

1) The CMB polarisation

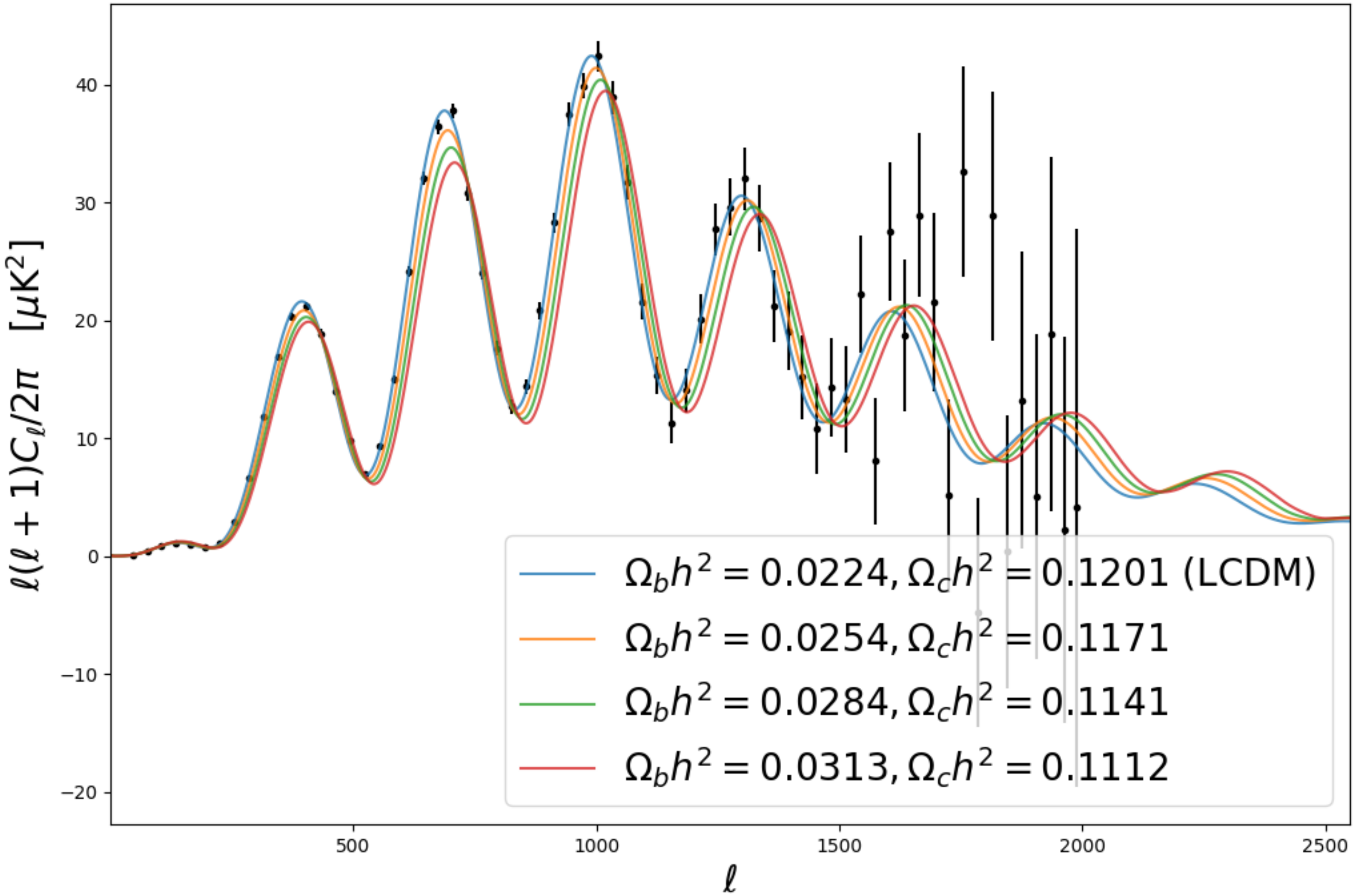


Just like for the temperature case, we can describe the polarisation properties with power spectra.



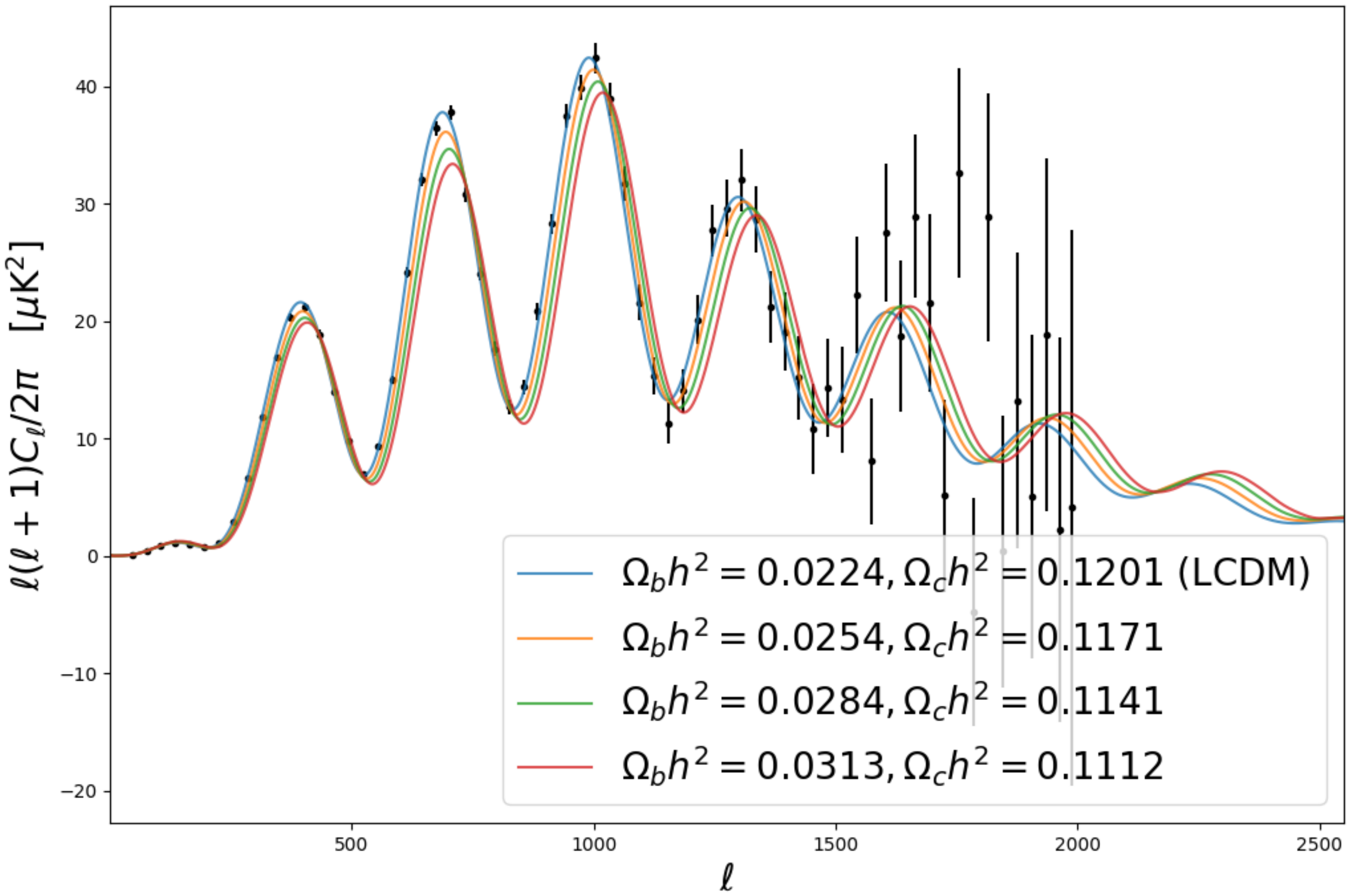
the fact that the temperature and polarisation data can be fitted by the same cosmological model is an important achievement, the polarisation results can be seen as a direct predictions once the temperature result was known

1) The CMB polarisation



In addition to the change of in peak position and damping effect, we see that polarisation amplitude decrease when we increase the baryon density.

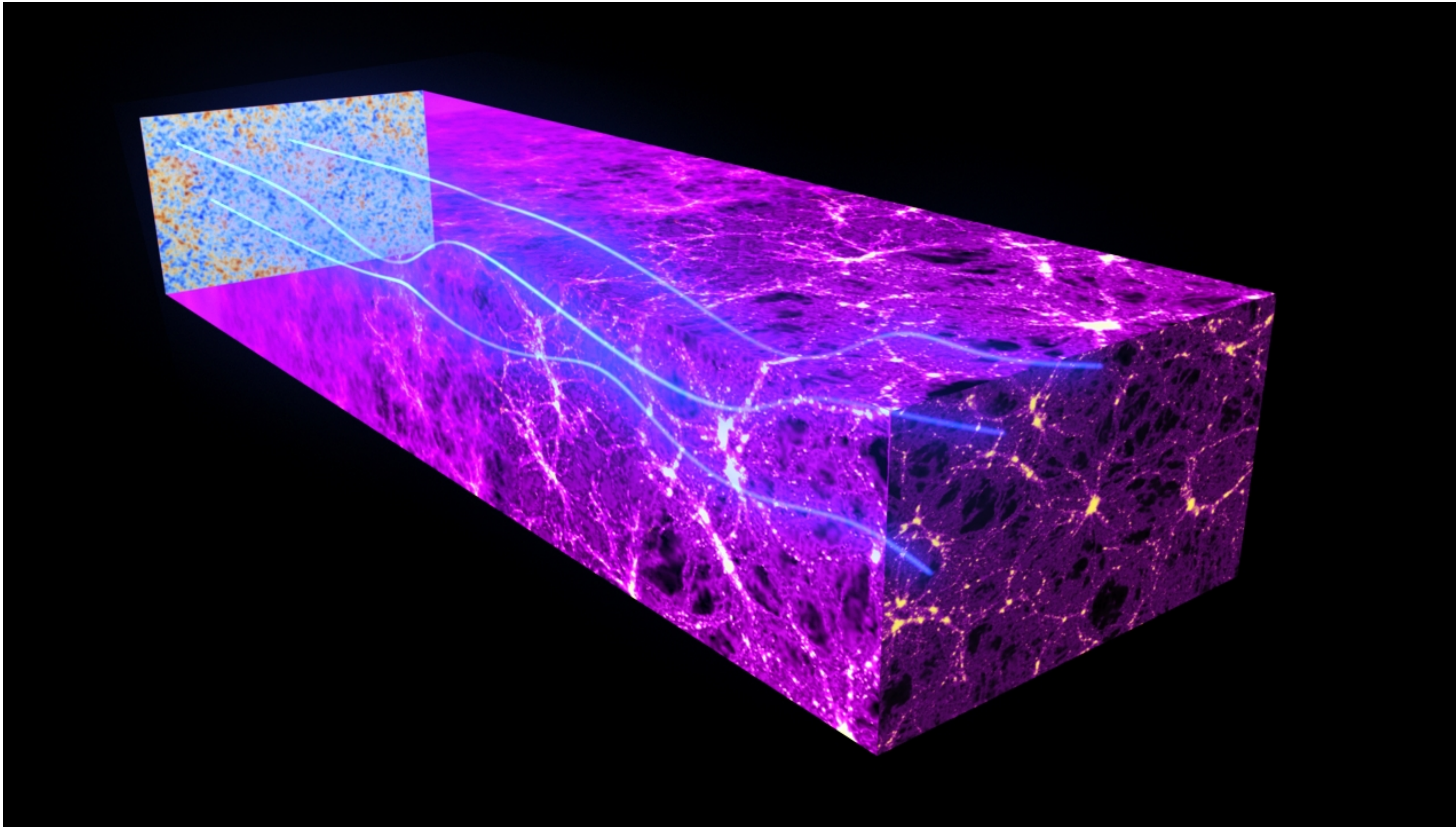
1) The CMB polarisation



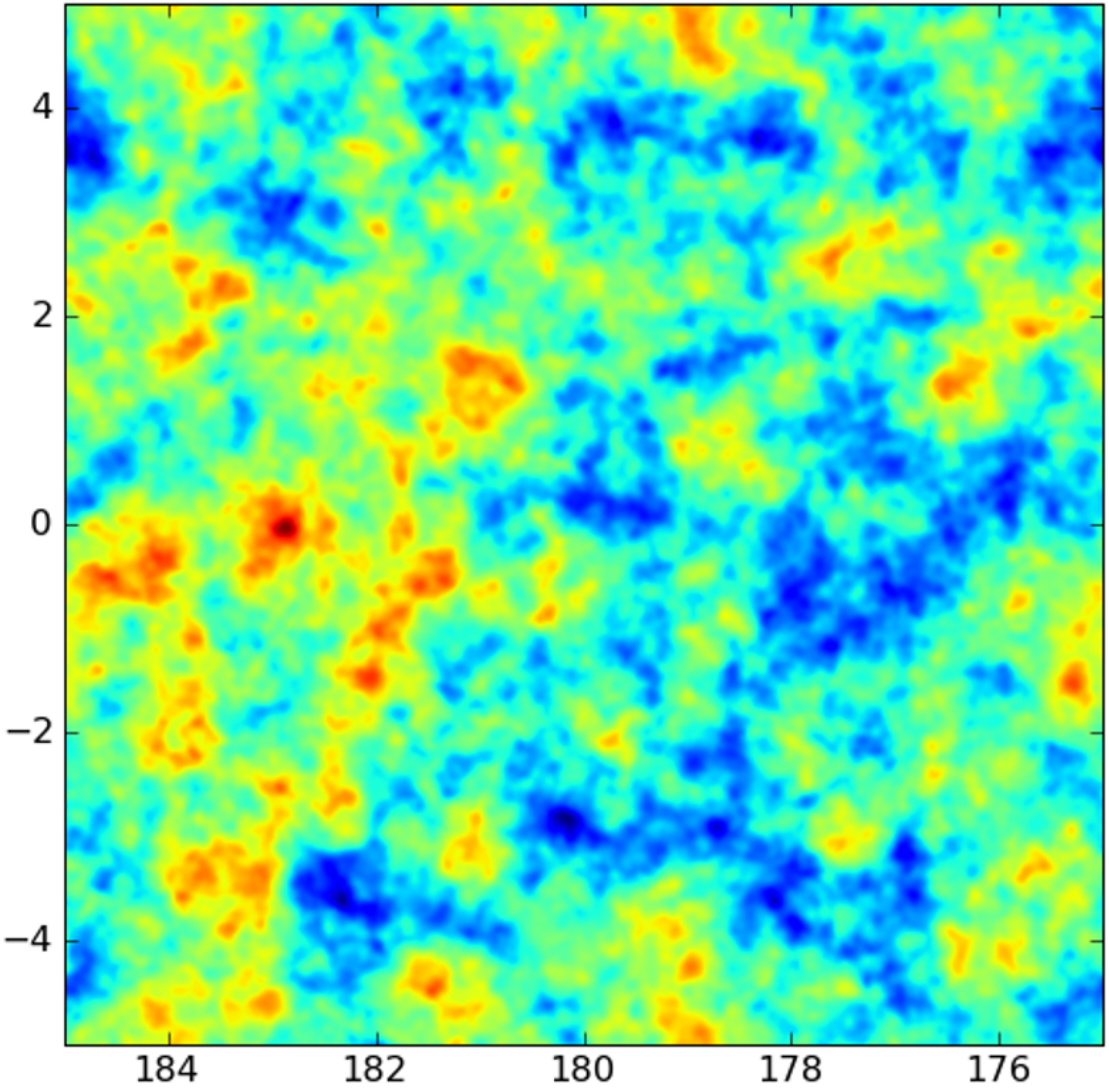
In addition to the change of in peak position and damping effect, we see that polarisation amplitude decrease when we increase the baryon density.

I think this is due to the fact that polarisation is sourced by quadrupole which itself depends on the mean free path of the photon, low baryon mean higher mean free path, and higher polarisation.

2) The CMB lensing

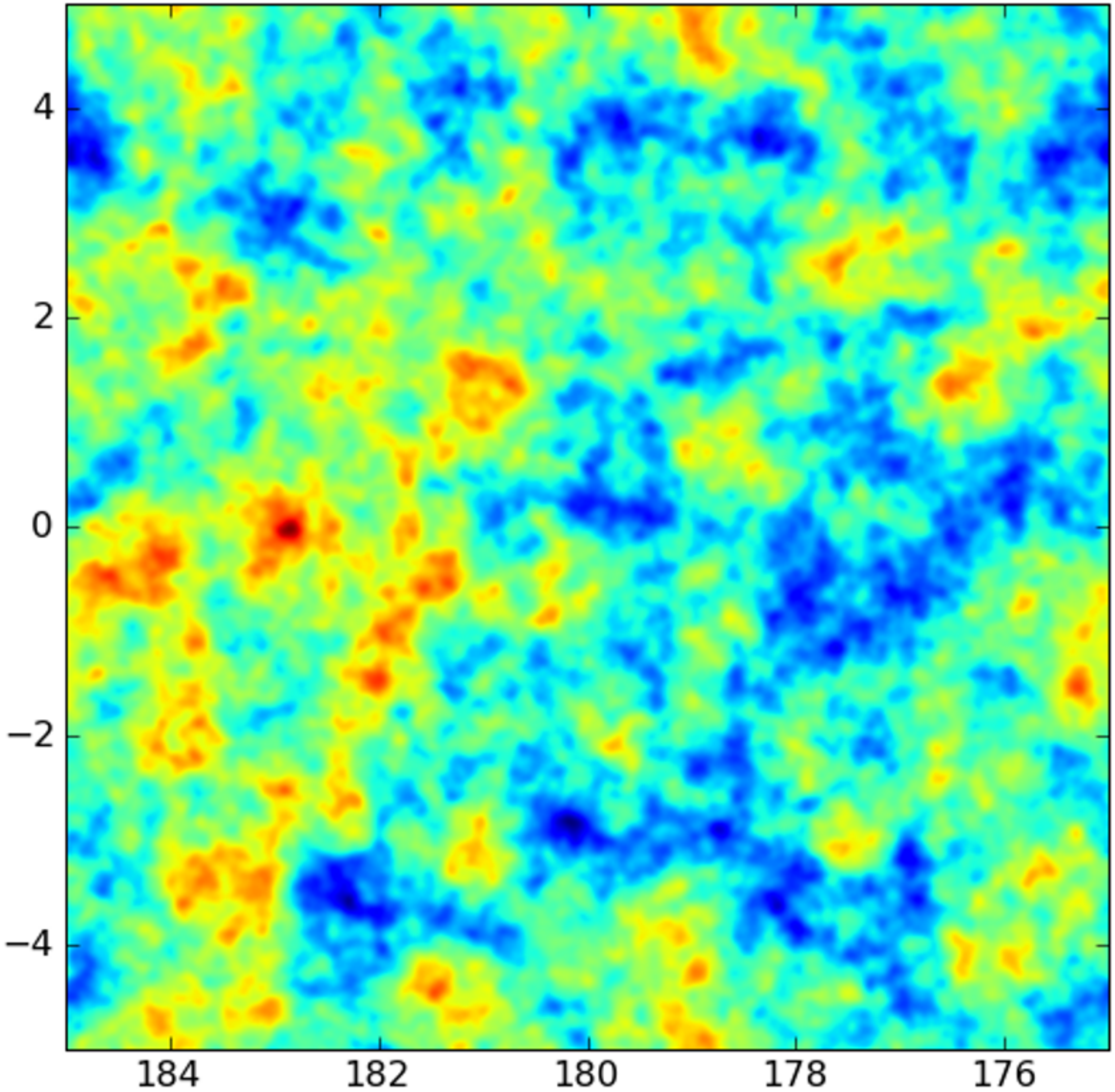


2) The CMB lensing



CMB temperature
(without lensing)

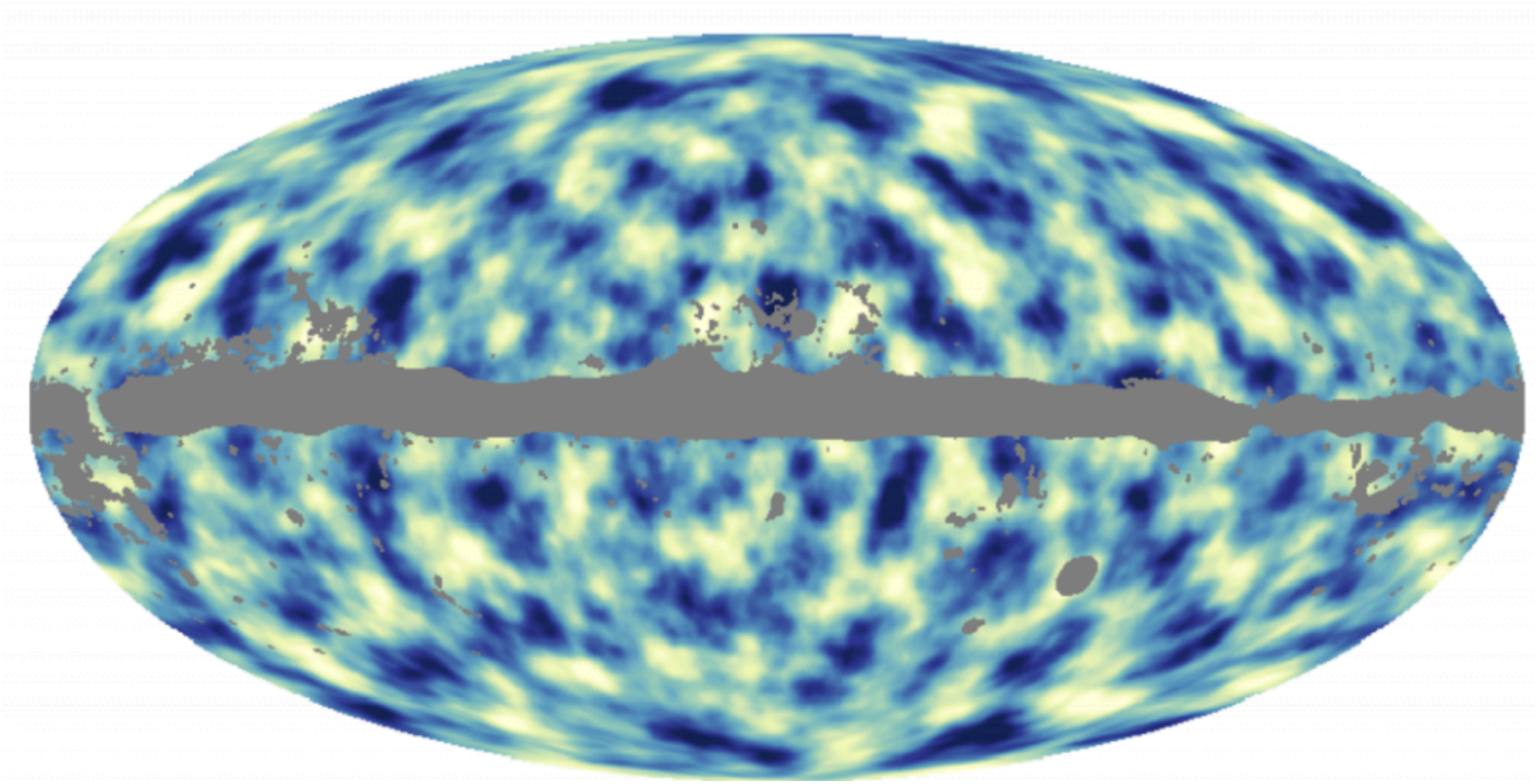
2) The CMB lensing



CMB temperature
(with lensing)

2) The CMB lensing

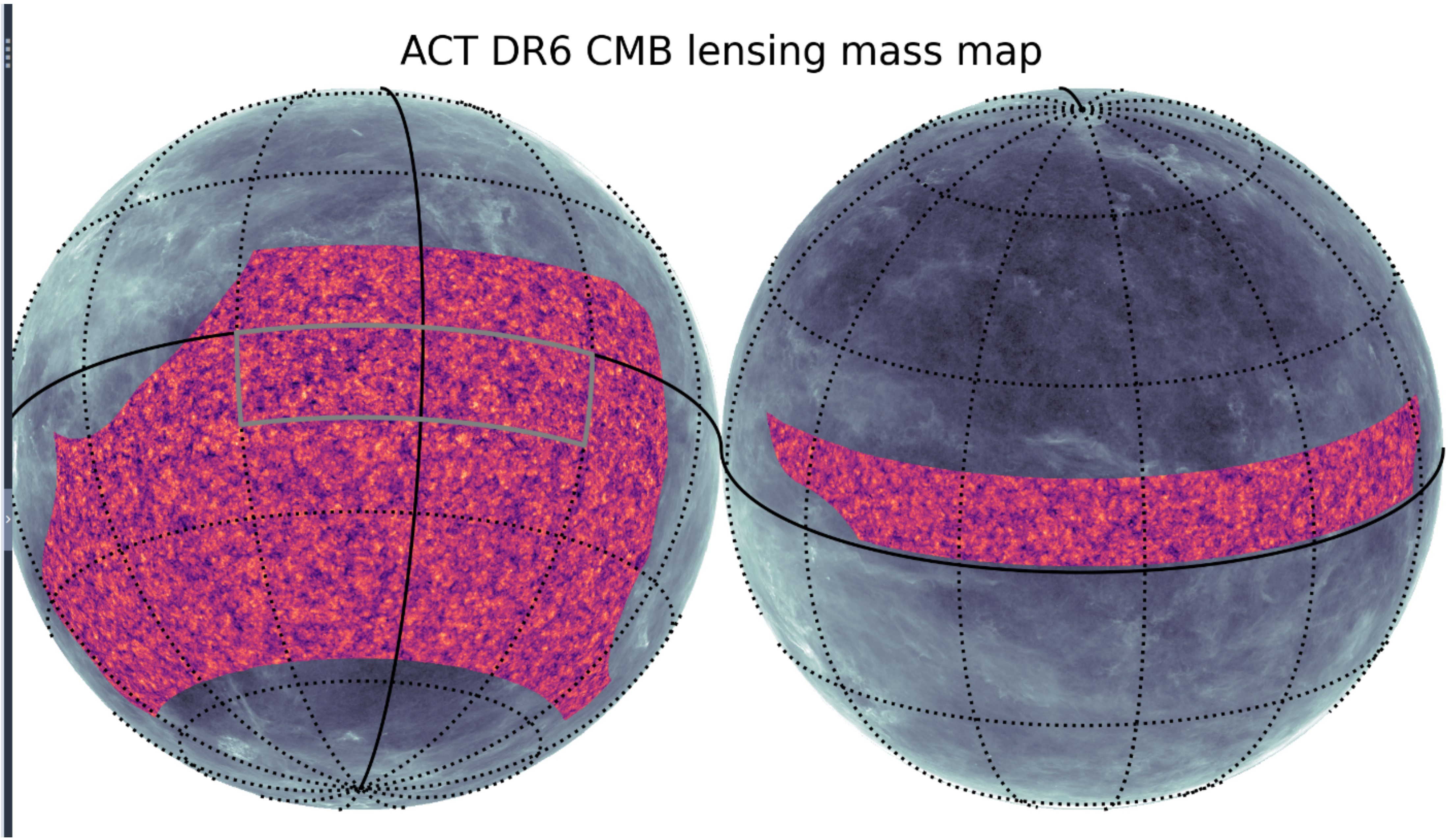
A full sky lensing map was reconstructed by the Planck satellite



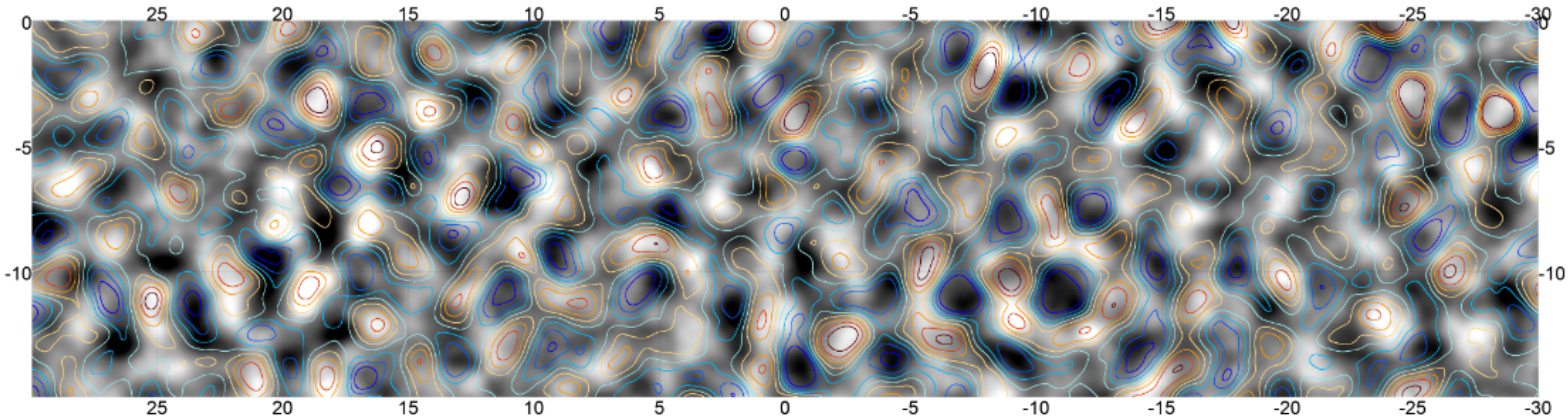
The S/N of this map is around 1,
Meaning that the features you see are
As likely to be noise than to be signal

2) The CMB lensing

Yesterday, the ACT collaboration presented a signal dominated reconstruction
Of CMB lensing



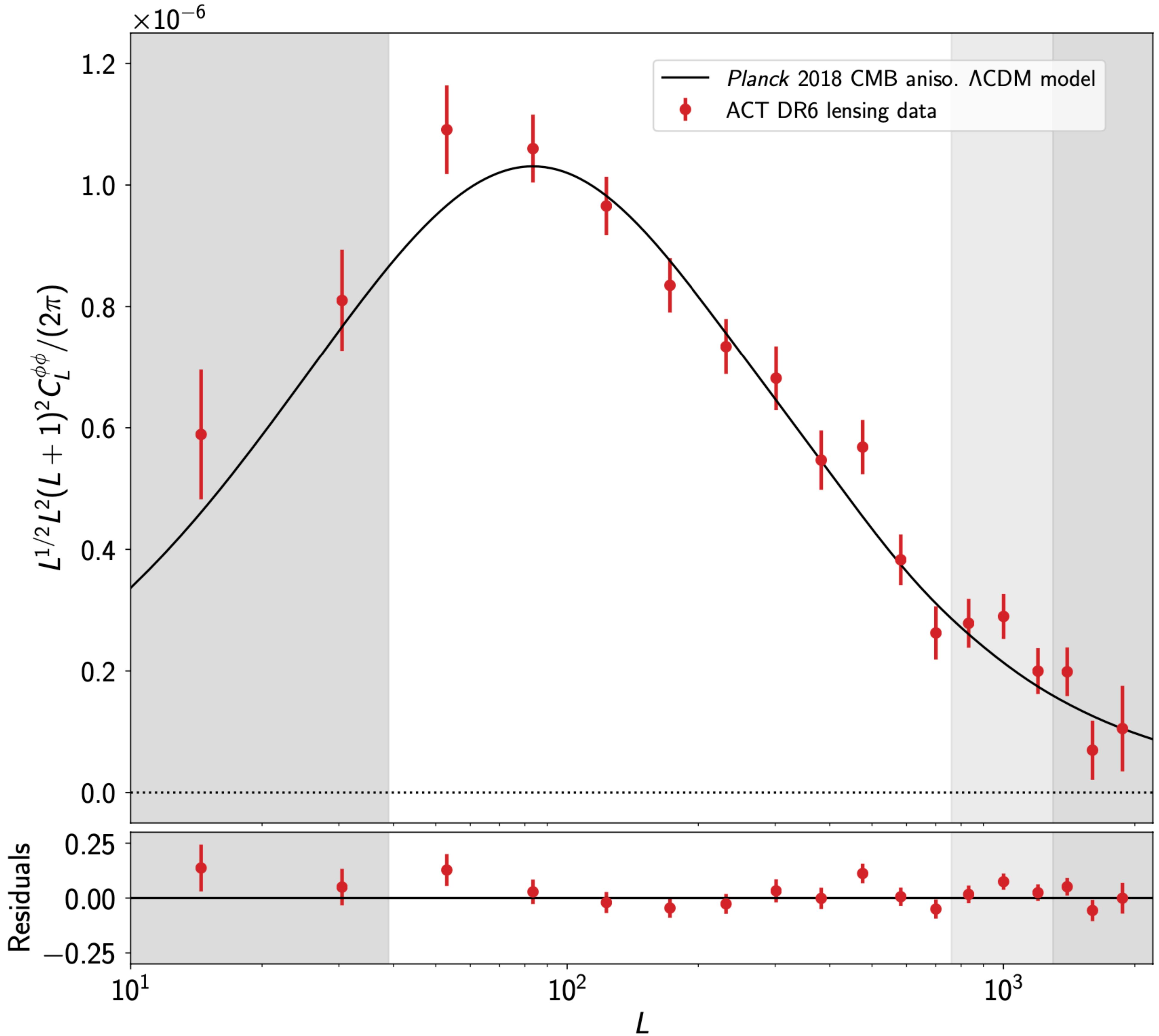
2) The CMB lensing

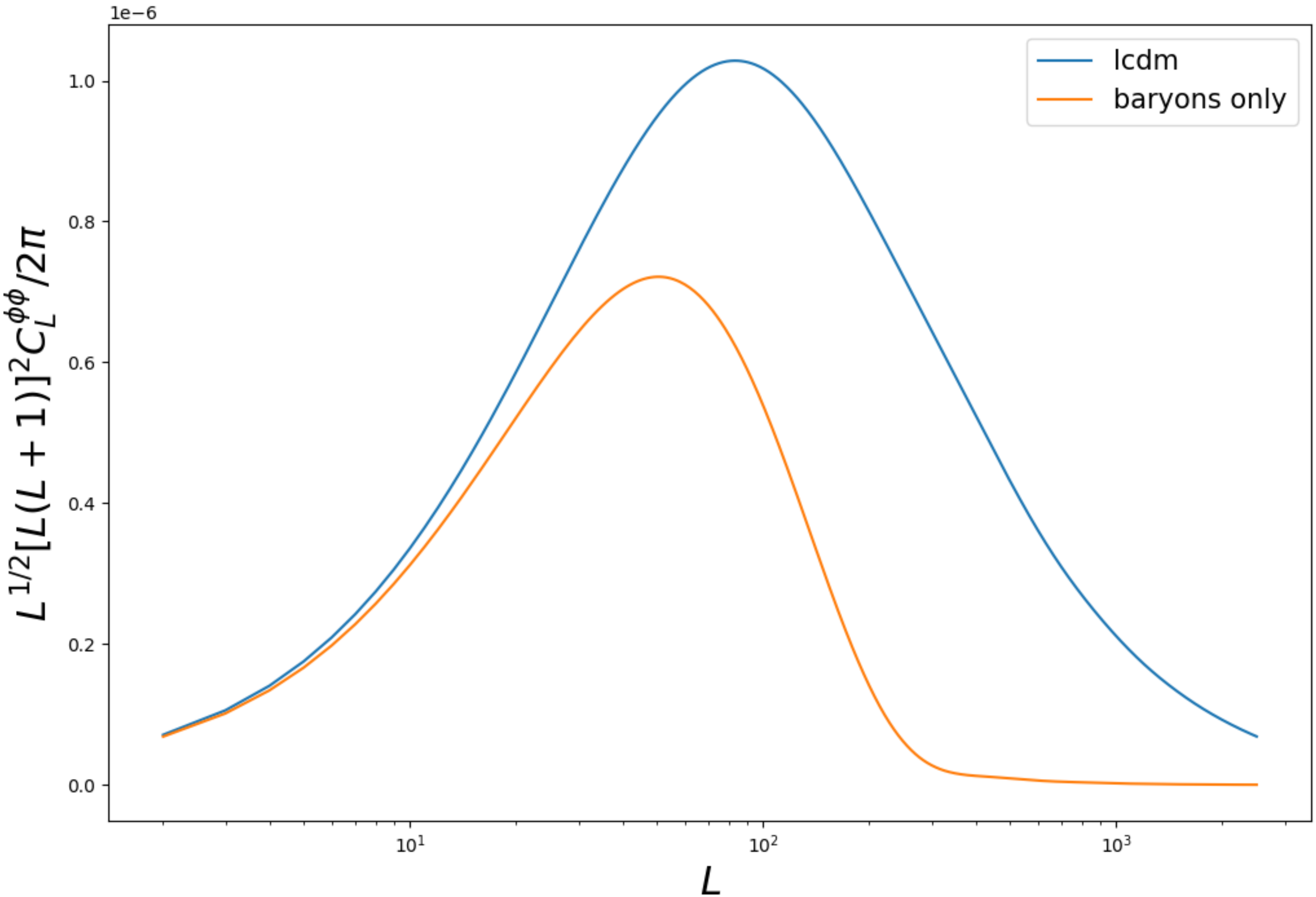


The features of this map really are the feature on the sky, here is a comparison of The lensing potential, in black and white, and overlaid, the emission from Young galaxy that make up the cosmic infrared background.

2) The CMB lensing

Just like for anything else, we can compute its power spectrum




1e−6
1.0
0.8
0.6
0.4
0.2
0.0
$L^{1/2}[L(L+1)]^2 C_L^{\phi\phi}/2\pi$
10^1
10^2
10^3
L
lcdm
baryons only

Thanks ! See you in 3 weeks (May 3rd)

$\mathcal{R}$ is called the curvature perturbation it is an interesting quantity that relates the initial perturbation in the inflaton field, to the initial perturbation in photons, baryons and dark matter.

$$\begin{aligned}
\psi_{\rm ini} &= -\frac{3}{2}\mathcal{R} \\
\delta_{\gamma,\rm ini} &= -2\psi_{\rm ini} \\
\delta_{b,\rm ini} &= \delta_{\rm dm,ini} = \frac{3}{4}\delta_{\gamma,\rm ini}
\end{aligned}$$

(These are called adiabatic initial conditions)

Using the initial conditions and the transfer functions, we can make predict the all of the statistical properties of our universe

We can expand Theta in Fourier space

$$a_{\ell m}(\boldsymbol{x},\eta) = \int d\hat{p}\ Y^*_{\ell m}(\hat{p})\Theta(\boldsymbol{x},\hat{p},\eta)$$

$$a_{\ell m}(\boldsymbol{x},\eta) = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\boldsymbol{x}} \int d\hat{p}\ Y^*_{\ell m}(\hat{p})\Theta(\boldsymbol{k},\hat{p},\eta)$$

And use the fact that we can solve the Boltzmann equation for Θ

$$\begin{aligned}
\Theta' + ik\mu\ \Theta &= -\phi' - ik\mu\psi - \tau'\left[\Theta_0 - \Theta + \mu u_b - \frac{1}{2}P_2\mu\Pi\right] \\
\delta_c' + iku_c &= -3\phi' \\
u_c' + \frac{a'}{a}u_c &= -ik\ \psi \\
\delta_b' + iku_b &= -3\phi' \\
u_b' + \frac{a'}{a}u_b &= -ik\ \psi + \frac{\tau'}{R}[u_b + 3i\Theta_1]
\end{aligned}$$

As in the matter power spectrum case, we can express the solution of the system of differential equations with the help of a transfer function

$$\Theta(\boldsymbol{k}, \hat{p}, \eta_R) = \mathcal{T}(k, \mu = \hat{k} \,.\, \hat{p})\mathcal{R}(\boldsymbol{k})$$

Note that unlike the case of matter, there is an angular dependence in the transfer function, here $\hat{p}$ is the photon propagation direction

The power spectrum of the observed CMB temperature anisotropies today is therefore given by

$$\begin{aligned}
\langle a_{\ell m}(\boldsymbol{x}, \eta_R) a^*_{\ell m}(\boldsymbol{x}, \eta_R)\rangle &= \langle \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\boldsymbol{x}} \int d\hat{p}\; Y^*_{\ell m}(\hat{p})\mathcal{T}(k, \hat{k} \,.\, \hat{p})\mathcal{R}(\boldsymbol{k}) \\
&\times \int \frac{d^3k'}{(2\pi)^3} e^{-i\boldsymbol{k'}\boldsymbol{x}} \int d\hat{p}'\; Y_{\ell m}(\hat{p}')\mathcal{T}^*(k, \hat{k}' \,.\, \hat{p}')\mathcal{R}(\boldsymbol{k'})\rangle
\end{aligned}$$

Using $\langle \mathcal{R}(\boldsymbol{k})\mathcal{R}(\boldsymbol{k'})\rangle = (2\pi)^3 \delta(\boldsymbol{k} - \boldsymbol{k'}) P_{\mathcal{R}}(k)$

We get :

$$\langle a_{\ell m}(\boldsymbol{x}, \eta_R) a^*_{\ell m}(\boldsymbol{x}, \eta_R)\rangle = \int \frac{d^3k}{(2\pi)^3} P_{\mathcal{R}}(k) \int d\hat{p}\; Y^*_{\ell m}(\hat{p})\mathcal{T}(k, \hat{k} \,.\, \hat{p}) \int d\hat{p}'\; Y_{\ell m}(\hat{p}')\mathcal{T}^*(k, \hat{k} \,.\, \hat{p}')$$

$$C_\ell = \int \frac{d^3k}{(2\pi)^3} P_{\mathcal{R}}(k) \int d\hat{p}\, Y^*_{\ell m}(\hat{p}) \mathcal{T}(k, \hat{k}\,.\,\hat{p}) \int d\hat{p}'\, Y_{\ell m}(\hat{p}) \mathcal{T}^*(k, \hat{k}\,.\,\hat{p}')$$

To go further we need to expand the transfer function into Legendre polynomial

$$\mathcal{T}(k, \hat{k}\,.\,\hat{p}) = \sum_\ell (-i)^\ell (2\ell+1) P_\ell(\hat{k}\,.\,\hat{p}) \mathcal{T}_\ell(k)$$

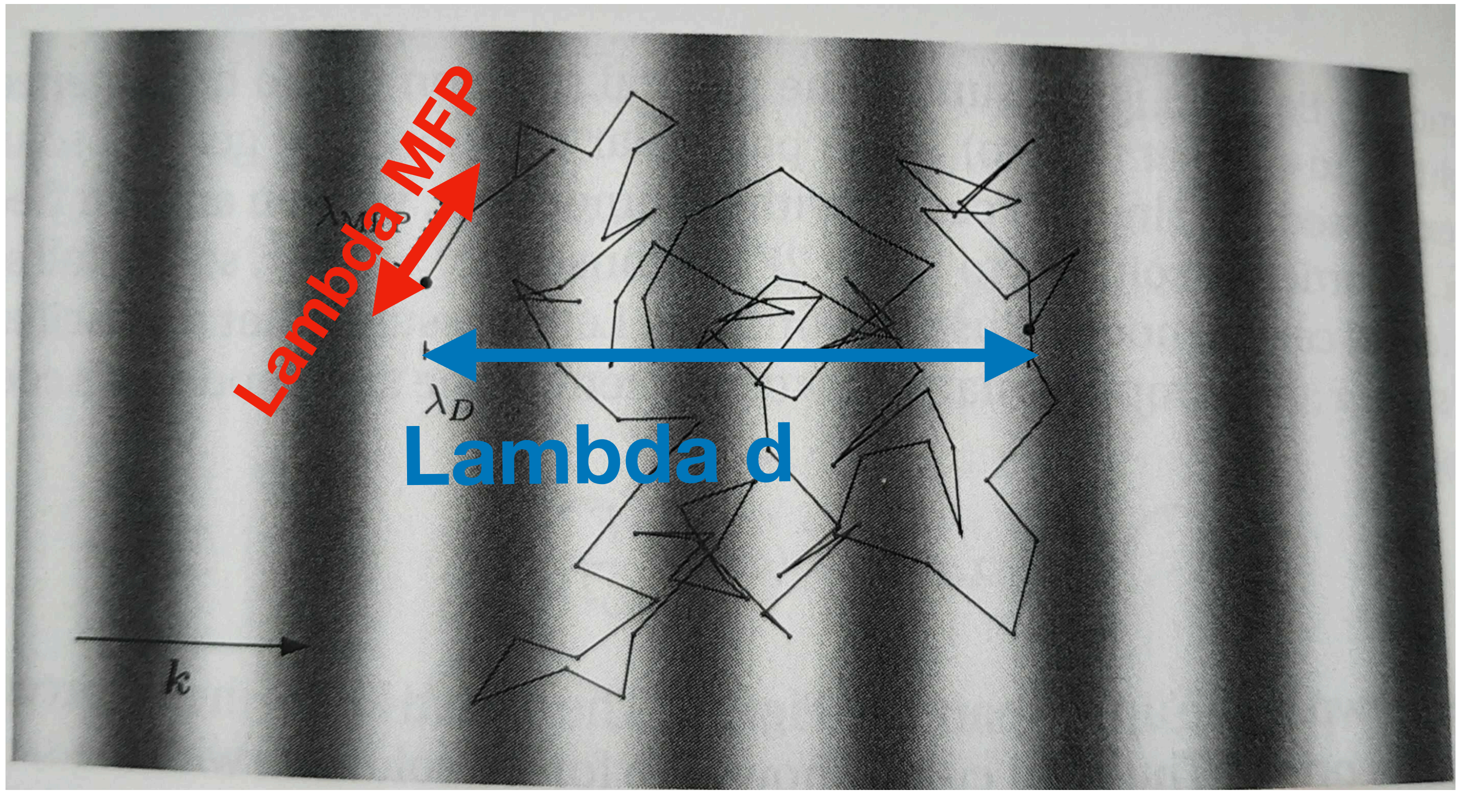
$$\begin{aligned} C_\ell &= \int \frac{d^3k}{(2\pi)^3} P_{\mathcal{R}}(k) \sum_{\ell', \ell''} (-i)^{\ell'} (i)^{\ell''} (2\ell'+1)(2\ell''+1) \mathcal{T}_{\ell'}(k) \mathcal{T}^*_{\ell''}(k) \\ &\times \int d\hat{p}\, Y^*_{\ell m}(\hat{p}) P_{\ell'}(\hat{k}\,.\,\hat{p}) \int d\hat{p}'\, Y_{\ell m}(\hat{p}') P_{\ell''}(\hat{k}\,.\,\hat{p}') \end{aligned}$$

$$\int d\hat{p}\, Y^*_{\ell m}(\hat{p}) P_{\ell'}(\hat{k}\,.\,\hat{p}) = \frac{4\pi Y^*_{\ell m}(\hat{k}) \delta_{\ell, \ell'}}{2\ell+1}$$

$$\begin{aligned} C_\ell &= \frac{2}{\pi} \int_0^\infty dk k^2 P_{\mathcal{R}}(k) |\mathcal{T}_\ell(k)|^2 \int d\hat{k} Y_{\ell m}(\hat{k}) Y^*_{\ell m}(\hat{k}) \\ &= \frac{2}{\pi} \int_0^\infty dk k^2 P_{\mathcal{R}}(k) |\mathcal{T}_\ell(k)|^2 \end{aligned}$$

2) When we increase the baryon density there is more power on small scales

Consider the path of a single photon as it scatters off a sea of electrons.



The physical mean free path of a photon is given by

$$\lambda_{\text{MFP,physical}} = (n_e \sigma_T)^{-1}$$

The comoving mean free path is given by $\lambda_{\text{MFP,comoving}} = (a n_e \sigma_T)^{-1}$

Over the course of one Hubble time, a photon scatter $n_e \sigma_T H^{-1}$ times

A cosmological photon therefore moves a mean comoving distance

$$\lambda_D \sim \lambda_{\text{MFP,comoving}} \sqrt{n_e \sigma_T H^{-1}} = \frac{1}{\sqrt{n_e \sigma_T H}} \frac{1}{a}$$

1) The CMB polarisation

So the way to generate polarisation on the last scattering is though Compton scattering of a bunch of photons on electrons, and the polarisation amplitude will be fully determined by the local quadrupole of the incoming radiation.

a) A second way to generate quadrupole on the last scattering surface is the hypothetical propagation of primordial gravitational waves

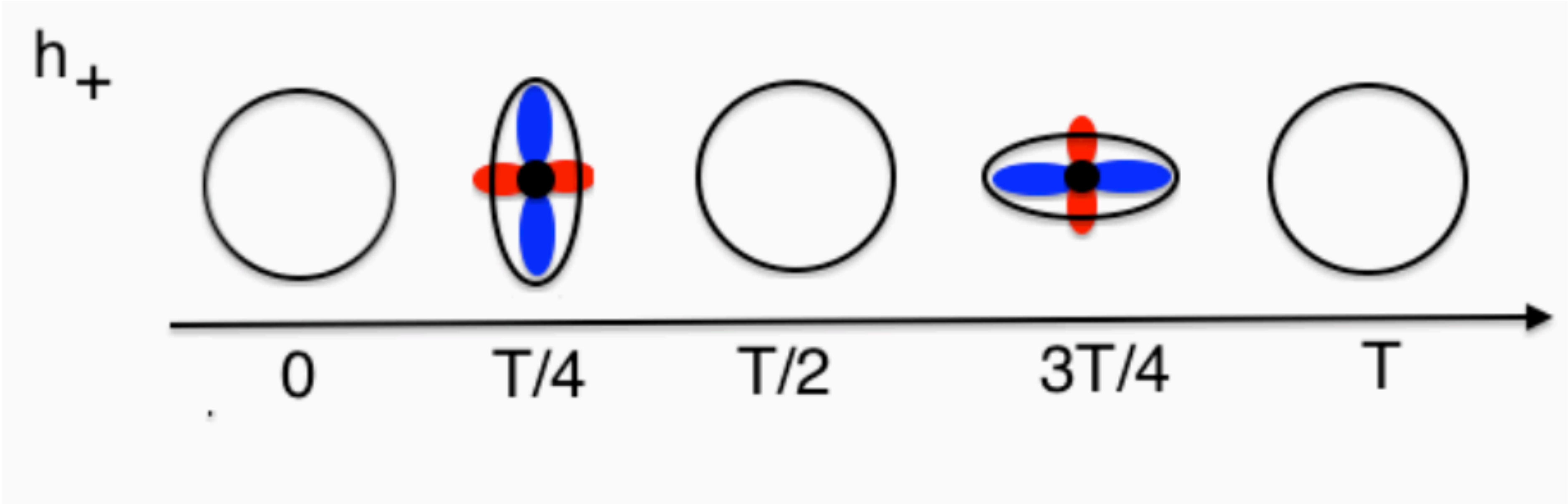


Figure 1.7: Effect of a gravitational wave on a circle of test particles. The generated quadrupole is another source of CMB polarization.